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General Energy Decay and Exponential Instability to a Nonlinear Dissipative-Dispersive Viscoelastic Petrovsky Equation

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Abstract. This work is concerned with the initial boundary value problem for a nonlinear viscoelastic Petrovsky wave equation

$$u_{tt} + \Delta^2 u - \int_0^t g(t-\tau)\Delta^2 u(\tau)d\tau - \Delta u_t - \Delta u_{tt} + u_t|u_t|^{m-1} = u|u|^{p-1}.$$

Under suitable conditions on the relaxation function g, the global existence of solutions is obtained without any relation between m and p. The uniform decay of solutions is proved by adapting the perturbed energy method. For $p > m$ and sufficient conditions on g, an unboundedness result of solutions is also obtained.

AMS Subject Classification: 35L35, 35B35, 35B40
Keywords and Phrases: Global existence, general decay, exponential growth, Petrovsky equation.

1 Introduction

In this paper, we investigate the problem

$$\begin{aligned} u_{tt} + \Delta^2 u - \int_0^t g(t-\tau)\Delta^2 u(\tau)d\tau - \Delta u_t \\ - \Delta u_{tt} + u_t|u_t|^{m-1} = u|u|^{p-1}, \quad x \in \Omega, t \geq 0, \end{aligned} \tag{1}$$

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$$u(x,t) = \partial_\nu u(x,t) = 0, \qquad x \in \partial\Omega, \quad t \geq 0, \tag{2}$$

$$u(x,0) = u_0, \quad u_t(x,0) = u_1, \qquad x \in \Omega, \tag{3}$$

where Ω is a bounded domain in $\mathrm{R}^n, n \geq 1$, with a smooth boundary $\partial\Omega$, $m, p > 1$, ν is the unit outer normal on $\partial\Omega$ and g is a non-negative function that represents the kernel of memory term.

In the absence of viscoelastic term ($g = 0$), the dispersive term Δu_{tt} and the strong damping dissipation Δu_t, problem (1)-(3) has been extensively studied and several results concerning existence, decay and nonexistence of solutions have been established. Generally, problems of the form

$$u_{tt} + \Delta^2 u + h(u_t) = f(u), \qquad x \in \Omega, \quad t > 0, \tag{4}$$

with the boundary and initial conditions (2) and (3), have been widely investigated by many authors. For $f = -q(x)u(x,t)$, equation (4) has been considered by Guesmia [5] where $q : \Omega \to \mathbb{R}^+$ is a positive function in $L^\infty(\Omega)$ and h is a continuous and increasing function which satisfies $h(0) = 0$. Using the semigroup approach, the author proved global existence, uniqueness and decay results under suitable growth conditions on h. When $h(u_t) = au_t|u_t|^{m-2}$ and $f(u) = bu|u|^{p-2}$ where $a, b > 0$ and $p, m > 2$, Messaoudi [10] established an existence result when $m \geq p$ with an arbitrary initial data while the solution blows up if $m < p$ and the initial energy is negative. The main point of the contribution is the method initiated by Gorgiev and Todorva [4] based on fixed point theorem. Related to this problem, Wu and Tsai [14] showed that the solution decays algebraically without the relation between m and p while it blows up in finite time if $p > m$ and the initial energy is nonnegative. In [2], Amroun and Benaissa obtained the global solvability of (4) subject to the same boundary and initial conditions as (2) and (3) where $f(u) = bu|u|^{p-2}$ and h satisfies

$$c_1|s| \leq |h(s)| \leq c_2|s|^r, \quad |s| \geq 1, \quad c_1, c_2 > 0,$$

under some appropriate restrictions on p and r. The key point to their proof is the use of stable set method combined with the Fadeo-Galerkin procedure.

Recently, in the presence of the strong damping, G. Li et al. [9] considered the following Petrovsky equation:

$$u_{tt} + \Delta^2 u - \Delta u_t + u_t|u_t|^{m-1} = u|u|^{p-1}, \qquad x \in \Omega, \quad t \geq 0, \tag{5}$$

with the boundary and initial conditions (2) and (3). The authors obtained the global existence and uniform decay of solutions if the initial data are in some stable set without any interaction between the damping mechanism $u_t|u_t|^{m-1}$ and the source term $u|u|^{p-1}$. Moreover, they established the blow up properties of local solution in the case $p > m$ and the initial energy is less than the potential well depth. In [18], for a wave equation (Δu instead of $\Delta^2 u$ in (5), S. Yu by using the stable set method showed that the solutions exist globally in time if $m \geq p$ and blow up in finite time if $m < p < \frac{2(m+1)}{n} + 1$.

There is a substantial number of papers concerning the study of nonlinear viscoelastic wave equations with the dispersive term Δu_{tt}. In the study of plates, Rivera et al. [13] considered the following viscoelastic equation

$$u_{tt} - \gamma \Delta u_{tt} + \Delta^2 u - \int_0^t g(t-\tau)\Delta^2 u(\tau)d\tau = 0.$$

They proved that the first and second order energy, associated with the solutions, decay exponentially provided the kernel of the memory also decays exponentially. For a related study, we may recall the work by Lagnese [8], who showed that the energy decays to zero as time goes to infinity by introducing a dissipative mechanism on the boundary of the system. In [3], Cavalvcanti et al. studied the global existence result and the uniform exponential decay of energy for the following equation:

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t-\tau)\Delta u(\tau)d\tau - \gamma \Delta u_t = 0 \tag{6}$$

In the case $\gamma = 0$, Messaoudi and Tatar [11] showed that the solution goes to zero with an exponential or polynomial rate under some restrictions on g. Using the potential well method, the same authors in [12] obtained global existence and an exponential decay result for an extension of (6) in the presence of nonlinear source term:

$$|u_t|^\rho u_{tt} - \Delta u - \gamma \Delta u_{tt} + \int_0^t g(t-\tau)\Delta u(\tau)d\tau - \Delta u_t = b|u|^{p-2}u.$$

Moreover, for sufficiently large values of the initial data and for a suitable relation between p and the relaxation function, they proved an unboundedness result. In [15], S.T. Wu proved the general decay of solutions for the nonlinear equation:

$$|u_t|^\rho u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + |u_t|^m u_t = |u|^p u. \qquad (7)$$

In [16], the author established the same result for equation (7) with the weak damping term ($m = 0$) and $p - 2$ instead of p. Without nonlinear source term, Han and Wang [6] obtained the general decay of solutions for (7) in the case $m = 0$. When $\rho = 0$ with $m - 2$ instead of m and in the absence of source term, they proved similar results in [7]. In the presence of dispersive term and strong damping term, in a recent work, R. Xu et al. [17] studied the global well-posedness for the wave equation

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau - \Delta u_{tt} - \Delta u_t + u_t = |u|^{p-1}u.$$

Defining a family of potential wells they proved existence and nonexistence of global solutions under some conditions with low initial energy while a blow up result is obtained with positive initial energy.

In the present work, our study will be devoted to the problem (1)-(3). Motivated by the above works, by introducing a suitable perturbed energy function, we study the asymptotic behavior of solution energy and we obtain the uniform decay of the energy under some assumptions on g without any interaction between source term and damping term. Under an appropriate restriction on g, we also prove that the solution exponentially grows when $m > p$ and the initial energy is negative.

The plan of this paper is as follows. In section 2, we introduce some notations and useful lemmas, and we state the local existence result Theorem 2.3. In section 3, we present the global existence result Lemma 3.2 and we show the exponential decay of the perturbed energy in Theorem 3.4. Growth properties of the problem (1)-(3) are given in Theorem 4.1.

2 Preliminaries

To prove our main results, we shall give some lemmas, assumptions and notations.

Lemma 2.1. *[1] (Sobolev-Poincaré inequality) Let q be a number with $2 \le q < \infty$ $(n = 1, 2, 3, 4)$ or $2 \le q \le \frac{2n}{n-4} (n \ge 5)$, then for $u \in H_0^2(\Omega)$ there is a constant $C_* = C(\Omega, q)$ such that*

$$\|u\|_q \le C_* \|\Delta u\|_2.$$

Assume that m and p satisfy

$$1 < p < \infty \quad (n = 1, 2, 3, 4) \quad \text{or} \quad 1 < p \le \frac{n}{n-4} \quad (n \ge 5), \tag{8}$$

$$1 < m < \infty \quad (n = 1, 2, 3, 4) \quad \text{or} \quad 1 < m \le \frac{n+4}{n-4} \quad (n \ge 5). \tag{9}$$

For the relaxation function we assume

(**G1**) $g \in C^1[0, \infty)$ is a non-negative and non-increasing function satisfying

$$1 - \int_0^\infty g(s)ds = l > 0, \quad g(0) > 0. \tag{10}$$

(**G2**) There exists a positive non-increasing differentiable function ξ such that

$$g'(t) \le -\xi(t) g(t), \quad \forall t \ge 0, \tag{11}$$

where $\int_0^{+\infty} \xi(t)dt = +\infty$.

Let us define the C^1 functionals $I, J, E : H_0^2(\Omega) \to \mathrm{R}$ by

$$I(t) = I(u(t)) = \left(1 - \int_0^t g(\tau)d\tau\right) \|\Delta u\|_2^2 + (g \circ \Delta u)(t) - \|u\|_{p+1}^{p+1}, \tag{12}$$

$$\begin{aligned} J(t) = J(u(t)) = \frac{1}{2}&\left(1 - \int_0^t g(\tau)d\tau\right) \|\Delta u\|_2^2 \\ &+ \frac{1}{2}(g \circ \Delta u)(t) - \frac{1}{p+1}\|u\|_{p+1}^{p+1}, \end{aligned} \tag{13}$$

$$E(t) = E(u(t)) = \frac{1}{2}\|u_t\|_2^2 + \frac{1}{2}\|\nabla u_t\|_2^2 + J(u), \tag{14}$$

where

$$(g \circ v)(t) = \int_0^t g(t-\tau)\int_\Omega |v(t) - v(\tau)|^2 dx d\tau,$$

and the stable set

$$W = \left\{u \in H_0^2(\Omega); I(u) > 0\right\} \cup \{0\}.$$

Lemma 2.2 *$E(t)$ is a non-increasing function for $t \geq 0$ and*

$$E'(t) = -\frac{1}{2}g(t)\|\Delta u\|_2^2 - \|\nabla u_t\|_2^2 + \frac{1}{2}(g' \circ \Delta u)(t) - \|u_t\|_{m+1}^{m+1}. \tag{15}$$

Proof. Multiplying (1) by u_t, integrating over Ω and using the boundary conditions, we get

$$E(t) - E(0) = -\int_0^t \Big(\frac{1}{2}g(t)\|\Delta u\|_2^2 + \|\nabla u_t\|_2^2 - \frac{1}{2}(g' \circ \Delta u)(t) + \|u_t\|_{m+1}^{m+1}\Big) dt.$$

Thus, the proof is completed.

We state a local existence theorem that can be established by combining the arguments of [10], [2] and [18].

Theorem 2.3 *Suppose that* (2.1), (2.2) *and* ($G1$) *hold and* $u_0, \in H_0^2(\Omega), u_1 \in H_0^1(\Omega)$. *Then there exists a unique weak solution $u(t)$ such that*

$$u \in C\left([0,T]; H_0^2(\Omega)\right) \cap C^1\left([0,T]; L^2(\Omega)\right),$$

$$u_t \in L^2\left([0,T]; H_0^1(\Omega)\right) \cap L^{m+1}(\Omega \times (0,T)),$$

for some positive constant T.

3 Global existence and energy decay

In this section we are going to establish a decay result for solutions of (1)-(3). For this purpose we use the Lyapunov functional method with suitable choice of a perturbed energy function. First, we present some lemmas that will be needed later.

Lemma 3.1 *Suppose that* (2.1) *and* $(G1)$ *hold. For any* $u_0 \in W$ *if*

$$\beta = \frac{C_*^{p+1}}{l}\left[\frac{2(p+1)}{l(p-1)}E(0)\right]^{\frac{p-1}{2}} < 1, \tag{16}$$

then $u(t) \in W$ *for all* $t \geq 0$.

Proof. Since $I(u_0) > 0$, then by continuity, there exists $T_* \leq T$ such that $I(u(t)) \geq 0$ for all $t \in [0, T_*)$. From (2.3), (2.5) ,(2.6) and the fact that $1 - \int_0^t g(\tau)d\tau > 1 - \int_0^\infty g(\tau)d\tau$, for all $t \in [0, T_*)$ we have

$$\begin{aligned} J(t) &= \frac{p-1}{2(p+1)}\left\{\left(1 - \int_0^t g(\tau)d\tau\right)\|\Delta u\|_2^2 + (g \circ \Delta u)(t)\right\} + \frac{1}{p+1}I(t) \\ &\geq \frac{p-1}{2(p+1)}\left\{\left(1 - \int_0^t g(\tau)d\tau\right)\|\Delta u\|_2^2 + (g \circ \Delta u)(t)\right\} \\ &\geq \frac{l(p-1)}{2(p+1)}\|\Delta u\|_2^2. \end{aligned} \tag{17}$$

Using (14), (17) and the lemma 2.2 we find

$$\|\Delta u\|_2^2 \leq \frac{2(p+1)}{l(p-1)}J(t) \leq \frac{2(p+1)}{l(p-1)}E(t) \leq \frac{2(p+1)}{l(p-1)}E(0), \tag{18}$$

for all $t \in [0, T_*)$. Then, by (16), (18) and the lemma 2.1 we obtain

$$\begin{aligned} \|u\|_{p+1}^{p+1} \leq C_*^{p+1}\|\Delta u\|_2^{p+1} &\leq C_*^{p+1}\left[\frac{2(p+1)}{l(p-1)}E(0)\right]^{\frac{p-1}{2}}\|\Delta u\|_2^2 \\ &= \beta l\|\Delta u\|_2^2 \leq \left(1 - \int_0^t g(\tau)d\tau\right)\|\Delta u\|_2^2, \end{aligned}$$

which implies $I(u(t)) > 0$ and so $u(t) \in W$ for all $t \in [0, T_*)$. By repeating this procedure, T_* can be extended to T.

Lemma 3.2 *Suppose that* (8), (9), $(G1)$ *and* (16) *hold. If* $u_0 \in W$, *then the solution of* (1)-(3) *is global and bounded.*

Proof. We use (12)-(14) and the lemma 2.2 to get

$$\begin{aligned} E(0) \geq E(t) &= \frac{1}{2}\|u_t\|_2^2 + \frac{1}{2}\|\nabla u_t\|_2^2 + J(t) \\ &= \frac{1}{2}\|u_t\|_2^2 + \frac{1}{2}\|\nabla u_t\|_2^2 + \frac{1}{p+1}I(t) \\ &+ \frac{p-1}{2(p+1)}\left\{\left(1 - \int_0^t g(\tau)d\tau\right)\|\Delta u\|_2^2 + (g \circ \Delta u)(t)\right\}. \end{aligned}$$

By the lemma 3.1, $I(t) \geq 0$. Using the assumption $G1$ we deduce

$$\|u_t\|_2^2 + \|\nabla u_t\|_2^2 + \|\Delta u\|_2^2 \leq CE(t) \leq CE(0), \qquad \forall t \geq 0,$$

where $C = \frac{2(p+1)}{l(p-1)}$. This shows that $\|u_t\|_2^2 + \|\nabla u_t\|_2^2 + \|\Delta u\|_2^2$ is uniformly bounded and independent of t for all $t \in [0, T)$. Therefore the solution of (1)-(3) is bounded and global in time.

By a suitable modification of the energy, we define

$$G(t) = ME(t) + \varepsilon\Psi(t) + \chi(t), \tag{19}$$

where M and ε are positive constants and

$$\Psi(t) = \int_\Omega (uu_t + \nabla u.\nabla u_t)dx + \frac{1}{2}\|\nabla u\|_2^2,$$

$$\chi(t) = \int_\Omega (\Delta u + \Delta u_t - u_t)\int_0^t g(t-\tau)\left(u(t) - u(\tau)\right)d\tau dx. \tag{20}$$

Lemma 3.3 *Suppose* $u_0 \in W$ *and* (16) *holds. Then for any solution of* (1)-(3) *there exists two positive constants* α_1 *and* α_2 *such that*

$$\alpha_1 E(t) \leq G(t) \leq \alpha_2 E(t), \tag{21}$$

for suitable choice of M and ε.

Proof. Using the Young's inequality, Lemma 2.1 and the Poincaré inequality, we find

$$\Psi(t) \leq \frac{1}{2}\|u_t\|_2^2 + \frac{1}{2}\|\nabla u_t\|_2^2 + \frac{1}{2}(C_*^2 + 2\lambda^{-1})\|\Delta u\|_2^2, \tag{22}$$

where λ is the Poincaré constant. By the Green's formula we have

$$\chi(t) = \int_\Omega (\Delta u - u_t) \int_0^t g(t-\tau)\,(u(t) - u(\tau))\,d\tau dx$$
$$- \int_\Omega \nabla u_t . \int_0^t g(t-\tau)\,(\nabla u(t) - \nabla u(\tau))\,d\tau dx.$$

We use the Young's inequality to obtain

$$\chi(t) \leq \frac{1}{2}\|u_t\|_2^2 + \frac{1}{2}\|\nabla u_t\|_2^2 + \frac{1}{2}\|\Delta u\|_2^2$$
$$+\frac{1}{2}\int_\Omega \left(\int_0^t g(t-\tau)(u(t)-u(\tau))d\tau\right)^2 dx$$
$$+\frac{1}{2}\int_\Omega \left(\int_0^t g(t-\tau)(\nabla u(t)-\nabla u(\tau))d\tau\right)^2 dx. \tag{23}$$

From the Hölder's inequality, lemma 2.1 and (10) we obtain

$$\int_\Omega \left(\int_0^t g(t-\tau)(u(t)-u(\tau))d\tau\right)^2 dx$$
$$\leq \int_\Omega \left(\int_0^t g(t-\tau)d\tau\right)\left(\int_0^t g(t-\tau)|u(t)-u(\tau)|^2 d\tau\right) dx$$
$$\leq (1-l)C_*^2(g\circ\Delta u)(t). \tag{24}$$

Similarly, by the Poincaré inequality we have

$$\int_\Omega \left(\int_0^t g(t-\tau)(\nabla u(t)-\nabla u(\tau))d\tau\right)^2 dx \leq (1-l)\lambda^{-1}(g\circ\Delta u)(t). \tag{25}$$

Inserting (24) and (25) into (23) and using (22), from (19) one can write

$$G(t) \leq ME(t) + \kappa_1 \|u_t\|_2^2 + \kappa_2 \|\nabla u_t\|_2^2 + \kappa_3 \|\Delta u\|_2^2 + \kappa_4 (g \circ \Delta u)(t),$$

where $\kappa_1 = \kappa_2 = \frac{1}{2}(1+\varepsilon)$, $\kappa_3 = \frac{1}{2}\left(\varepsilon(C_*^2 + 2\lambda^{-1}) + 1\right)$ and $\kappa_4 = \frac{1}{2}(1-l)(C_*^2 + \lambda^{-1})$. Then, by (14), the lemma 3.2 and choosing ε small enough and M sufficiently large, there exists a positive constant $\overline{\alpha}_1$ such that $G(t) \leq \overline{\alpha}_1 E(t)$. By the same method, we can show that $G(t) \geq \overline{\alpha}_2 E(t)$ for some positive constant $\overline{\alpha}_2$. This completes the proof.

Now, we state our main result.

Theorem 3.4 *Let $u_0 \in W$ be given which satisfies (16). Suppose that (8), (9), $(G1)$ and $(G2)$ hold. Then, for each $t_0 > 0$, there exists positive constants k and κ such that the global solution of (1)-(3) satisfies*

$$E(t) \leq Ke^{-\kappa \int_{t_0}^{t} \xi(s)ds}, \qquad \forall t \geq t_0. \tag{26}$$

To prove the theorem 3.4, we need to establish the following lemmas.

Lemma 3.5 *Let $u_0 \in W$ be given and satisfying (16). Suppose that $G1$ holds. If u is the solution of (1)-(3), then there exists positive constants k_1 and k_2 such that*

$$\Psi'(t) \leq \|u_t\|_2^2 + \|\nabla u_t\|_2^2 - \frac{l}{3}\|\Delta u\|_2^2 + k_1 (g \circ \Delta u)(t) + k_2 \|u_t\|_{m+1}^{m+1} + \|u\|_{p+1}^{p+1}.$$

Proof. Taking the derivative of $\Psi(t)$ and using (1), it follows that

$$\Psi'(t) = \|u_t\|_2^2 + \|\nabla u_t\|_2^2 - \|\Delta u\|_2^2 + \|u\|_{p+1}^{p+1} + \int_\Omega \int_0^t g(t-\tau)\Delta u(t)\Delta u(\tau) d\tau dx - \int_\Omega u u_t |u_t|^{m-1} dx. \tag{27}$$

By the Young's inequality and (10), for the fifth term of the right-hand side of (27), for any $\eta > 0$ we obtain

$$\left| \int_\Omega \int_0^t g(t-\tau)\Delta u(t)\Delta u(\tau) d\tau dx \right|$$

$$\leq \int_\Omega \left(\int_0^t g(t-\tau)|\Delta u(\tau) - \Delta u(t)||\Delta u(t)| d\tau \right) dx + \int_0^t g(\tau) d\tau \|\Delta u\|_2^2$$
$$\leq (1+\eta) \int_0^t g(\tau) d\tau \|\Delta u\|_2^2 + \frac{1}{4\eta}(g \circ \Delta u)(t)$$
$$\leq (1+\eta)(1-l)\|\Delta u\|_2^2 + \frac{1}{4\eta}(g \circ \Delta u)(t). \tag{28}$$

For the last integral of the right-hand side of (27), we use Young's inequality and lemmas 2.1 and 3.1 to get

$$\left| - \int_\Omega u u_t |u_t|^{m-1} dx \right| \leq \eta \|u\|_{m+1}^{m+1} + c(\eta)\|u_t\|_{m+1}^{m+1}$$
$$\leq \eta c_1 \|\Delta u\|_2^2 + c(\eta)\|u_t\|_{m+1}^{m+1}, \tag{29}$$

where $c_1 = C_*^{m+1} \left(\frac{2(p+1)}{l(p-1)} E(0) \right)^{\frac{m-1}{2}}$. Inserting (28) and (29) into (27), we arrive at

$$\Psi'(t) \leq \|u_t\|_2^2 + \|\nabla u_t\|_2^2 + (\eta c_1 + (1+\eta)(1-l) - 1) \|\Delta u\|_2^2$$
$$+ \frac{1}{4\eta}(g \circ \Delta u)(t) + c(\eta)\|u_t\|_{m+1}^{m+1} + \|u\|_{p+1}^{p+1}.$$

Now taking $\eta = \frac{2l}{3(c_1+1-l)}$, we obtain the result.

Lemma 3.6 *Let $u_0 \in W$ be given and satisfying (16). Suppose that G1 holds. If u is the solution of (1)-(3), then there exists positive constants k_3, k_4, k_5 and k_6 such that for all $\gamma > 0$ we have*

$$\chi'(t) \leq \left(\gamma - \int_0^t g(\tau) d\tau \right) \|u_t\|_2^2 + \left(\frac{k_3}{\gamma} + \gamma - \int_0^t g(\tau) d\tau \right) \|\nabla u_t\|_2^2$$
$$+ \gamma k_4 \|\Delta u\|_2^2 + \varphi(\gamma)(g \circ \Delta u)(t) - \frac{k_5}{\gamma}(g' \circ \Delta u)(t) + \gamma k_6 \|u_t\|_{m+1}^{m+1}, \tag{30}$$

where φ is a positive function of γ that will be given in the proof.

Proof. Differentiate (20) with respect to t and using (1), we get

$$\chi'(t) = \int_\Omega \Delta u \int_0^t g(t-\tau)(\Delta u(t) - \Delta u(\tau)) d\tau dx$$

$$
\begin{aligned}
&-\int_\Omega \left(\int_0^t g(t-\tau)\Delta u(\tau)d\tau\right)\left(\int_0^t g(t-\tau)(\Delta u(t)-\Delta u(\tau))d\tau\right)dx\\
&+\int_\Omega u_t|u_t|^{m-1}\int_0^t g(t-\tau)(u(t)-u(\tau))d\tau dx\\
&-\int_\Omega u|u|^{p-1}\int_0^t g(t-\tau)(u(t)-u(\tau))d\tau dx\\
&+\int_\Omega \Delta u\int_0^t g'(t-\tau)(u(t)-u(\tau))d\tau dx\\
&-\int_\Omega u_t\int_0^t g'(t-\tau)(u(t)-u(\tau))d\tau dx\\
&-\int_\Omega \nabla u_t\int_0^t g'(t-\tau)(\nabla u(t)-\nabla u(\tau))d\tau dx\\
&+\left(\int_\Omega u_t\Delta u dx-\|\nabla u_t\|_2^2-\|u_t\|_2^2\right)\int_0^t g(\tau)d\tau.
\end{aligned}
\tag{31}
$$

Next, we will estimate terms on the right-hand side of (31).
For the first term we have

$$
\begin{aligned}
&\int_\Omega \Delta u\int_0^t g(t-\tau)(\Delta u(t)-\Delta u(\tau))d\tau dx\\
&\qquad\le \gamma\|\Delta u\|_2^2+\frac{1}{4\gamma}\int_\Omega\left(\int_0^t g(t-\tau)(\Delta u(t)-\Delta u(\tau))d\tau\right)^2dx\\
&\qquad\le \gamma\|\Delta u\|_2^2+\frac{1}{4\gamma}(1-l)(g\circ\Delta u)(t),
\end{aligned}
\tag{32}
$$

where γ is an arbitrary positive constant. For the second term we obtain

$$
\begin{aligned}
&\left|-\int_\Omega \left(\int_0^t g(t-\tau)\Delta u(\tau)d\tau\right)\left(\int_0^t g(t-\tau)(\Delta u(t)-\Delta u(\tau))d\tau\right)dx\right|\\
&\quad\le \gamma\int_\Omega\Big(\int_0^t g(t-\tau)\Delta u(\tau)d\tau\Big)^2dx\\
&\qquad\qquad+\frac{1}{4\gamma}\int_\Omega\Big(\int_0^t g(t-\tau)(\Delta u(t)-\Delta u(\tau))d\tau\Big)^2dx
\end{aligned}
$$

$$\leq \gamma \int_{\Omega} \left(\int_{0}^{t} g(t-\tau)\Delta u(\tau) d\tau \right)^{2} dx + \frac{1}{4\gamma}(1-l)(g \circ \Delta u)(t). \tag{33}$$

The first integral in the right hand-side of (33) can be estimated in the form

$$\int_{\Omega} \left(\int_{0}^{t} g(t-\tau)\Delta u(\tau) d\tau \right)^{2} dx$$

$$\leq \int_{\Omega} \left(\int_{0}^{t} g(t-\tau)(|\Delta u(\tau) - \Delta u(t)| + |\Delta u(t)|) d\tau \right)^{2} dx$$

$$\leq 2 \int_{\Omega} \Big(\int_{0}^{t} g(t-\tau)|\Delta u(\tau) - \Delta u(t)| d\tau \Big)^{2} dx$$

$$+ 2 \int_{\Omega} \Big(\int_{0}^{t} g(t-\tau)|\Delta u(t)| d\tau \Big)^{2} dx$$

$$\leq 2(1-l)(g \circ \Delta u)(t) + 2(1-l)^{2}\|\Delta u\|_{2}^{2}. \tag{34}$$

By (34), for the inequality (33) we have

$$\left| -\int_{\Omega} \left(\int_{0}^{t} g(t-\tau)\Delta u(\tau) d\tau \right) \left(\int_{0}^{t} g(t-\tau)(\Delta u(t) - \Delta u(\tau)) d\tau \right) dx \right|$$

$$\leq (2\gamma + \frac{1}{4\gamma})(1-l)(g \circ \Delta u)(t) + 2\gamma(1-l)^{2}\|\Delta u\|_{2}^{2}, \tag{35}$$

We use Young's inequality, lemmas 2.1 and 3.2 to estimate the third term as

$$\int_{\Omega} u_{t}|u_{t}|^{m-1} \int_{0}^{t} g(t-\tau)(u(t)-u(\tau)) d\tau dx$$

$$\leq \int_{0}^{t} g(t-\tau) \left(\gamma\|u_{t}\|_{m+1}^{m+1} + c(\gamma)\|u(t) - u(\tau)\|_{m+1}^{m+1} \right) d\tau$$

$$\leq \gamma(1-l)\|u_{t}\|_{m+1}^{m+1} + c(\gamma)C_{*}^{m+1} \int_{0}^{t} g(t-\tau)\|\Delta u(t) - \Delta u(\tau)\|_{2}^{m+1} d\tau$$

$$\leq \gamma(1-l)\|u_{t}\|_{m+1}^{m+1} + c(\gamma)c_{2}(g \circ \Delta u)(t), \tag{36}$$

where $c_2 = C_*^{m+1}\left(\frac{2(p+1)}{l(p-1)}E(0)\right)^{\frac{m-1}{2}}$. For the fourth term we have

$$\begin{aligned}
&\int_\Omega u|u|^{p-1}\int_0^t g(t-\tau)(u(t)-u(\tau))d\tau dx\\
&\le \gamma\int_\Omega |u|^{2p}dx+\frac{1}{4\gamma}\int_\Omega\left(\int_0^t g(t-\tau)(u(t)-u(\tau))d\tau\right)^2 dx\\
&\le \gamma C_*^{2p}\|\Delta u\|_2^{2p}+\frac{1}{4\gamma}C_*^2(1-l)(g\circ\Delta u)(t)\\
&\le \gamma c_3\|\Delta u\|_2^2+\frac{1}{4\gamma}C_*^2(1-l)(g\circ\Delta u)(t),
\end{aligned} \tag{37}$$

where $c_3 = C_*^{2p}\left(\frac{2(p+1)}{l(p-1)}E(0)\right)^{p-1}$. Concerning the fifth and sixth terms, we get

$$\int_\Omega \Delta u\int_0^t g'(t-\tau)(u(t)-u(\tau))d\tau dx \le \gamma\|\Delta u\|_2^2-\frac{1}{4\gamma}g(0)C_*^2(g'\circ\Delta u)(t), \tag{38}$$

$$\int_\Omega u_t\int_0^t g'(t-\tau)(u(t)-u(\tau))d\tau dx \le \gamma\|u_t\|_2^2-\frac{1}{4\gamma}g(0)C_*^2(g'\circ\Delta u)(t). \tag{39}$$

We use the Young and Poincaré inequalities to obtain

$$\begin{aligned}
&\left|-\int_\Omega \nabla u_t\int_0^t g'(t-\tau)(\nabla u(t)-\nabla u(\tau))d\tau dx\right|\\
&\qquad\le \gamma\|\nabla u_t\|_2^2+\frac{1}{4\gamma}\int_\Omega\left(\int_0^t g'(t-\tau)(\nabla u(t)-\nabla u(\tau))d\tau\right)^2 dx\\
&\qquad\le \gamma\|\nabla u_t\|_2^2-\frac{g(0)}{4\gamma}\int_\Omega\int_0^t g'(t-\tau)|\nabla u(t)-\nabla u(\tau)|^2 d\tau dx\\
&\quad\le \gamma\|\nabla u_t\|_2^2-\frac{g(0)}{4\gamma}\lambda^{-1}(g'\circ\Delta u)(t),
\end{aligned} \tag{40}$$

where λ is the Poincaré constant. Again applying Young and Poincaré inequalities, for the last three terms in the right-hand side of (31) we find

$$\left(\int_\Omega u_t\Delta u dx-\|\nabla u_t\|_2^2-\|u_t\|_2^2\right)\int_0^t g(\tau)d\tau$$

$$\leq \left(\frac{1-l}{4\gamma} - \int_0^t g(\tau)d\tau\right) \|u_t\|_2^2 + \gamma(1-l)\|\Delta u\|_2^2 - \int_0^t g(\tau)d\tau \|\nabla u_t\|_2^2$$
$$\leq - \int_0^t g(\tau)d\tau \|u_t\|_2^2 + \gamma(1-l)\|\Delta u\|_2^2 + \left(\frac{1-l}{4\gamma}\lambda^{-1} - \int_0^t g(\tau)d\tau\right) \|\nabla u_t\|_2^2. \tag{41}$$

Combining (32) and (35)-(41), the inequality (30) follows with $k_3 = \frac{1-l}{4}\lambda^{-1}$, $k_4 = 2l^2 - 5l + 5 + c_3$, $k_5 = \frac{g(0)}{2}(C_*^2 + \frac{\lambda^{-1}}{2})$, $k_6 = 1 - l$ and

$$\varphi(\gamma) = \frac{1}{4\gamma}\left[(1-l)(2 + 8\gamma^2 + C_*^2) + 4\gamma c(\gamma)c_2\right].$$

At this point, we are ready to prove Theorem 3.4.

Proof of Theorem 3.4. The assumption (G1) guarantees that for any $t_0 > 0$ we have

$$\int_0^t g(\tau)d\tau \geq \int_0^{t_0} g(\tau)d\tau = g_0, \tag{42}$$

for all $t \geq t_0$. Therefore, from (19), (15), (42) and the lemmas 3.5, 3.6 we obtain

$$G'(t) \leq -(g_0 - \varepsilon - \gamma)\|u_t\|_2^2 - \left(M + g_0 - \varepsilon - \gamma - \frac{k_3}{\gamma}\right) \|\nabla u_t\|_2^2$$
$$- \left(\frac{\varepsilon l}{3} - \gamma k_4\right) \|\Delta u\|_2^2 - (M - \varepsilon k_2 - \gamma k_6)\|u_t\|_{m+1}^{m+1}$$
$$+ \left(\frac{M}{2} - \frac{k_5}{\gamma}\right)(g' \circ \Delta u)(t) + (\varepsilon k_1 + \varphi(\gamma))(g \circ \Delta u)(t) + \varepsilon\|u\|_{p+1}^{p+1}. \tag{43}$$

Now, we take $\varepsilon < \frac{g_0}{2}$ and $\gamma > 0$ sufficiently small such that

$$\gamma < \min\left\{g_0 - \varepsilon, \frac{\varepsilon l}{3k_4}\right\}.$$

Whence ε and γ are fixed, we choose M large enough that

$$M > \max\left\{\gamma + \frac{k_3}{\gamma}, \varepsilon k_2 + \gamma k_6, \frac{2k_5}{\gamma}\right\}.$$

Hence, (43) implies that there exist positive constants μ_1 and μ_2 such that

$$G'(t) \leq -\mu_1 E(t) + \mu_2 (g \circ \Delta u)(t), \tag{44}$$

for all $t \geq t_0$. Multiplying (44) by $\xi(t)$, using (11) and (15), we get

$$\begin{aligned}\xi(t)G'(t) &\leq -\mu_1\xi(t)E(t) + \mu_2\xi(t)(g \circ \Delta u)(t) \\ &\leq -\mu_1\xi(t)E(t) - \mu_2(g' \circ \Delta u)(t) \\ &\leq -\mu_1\xi(t)E(t) - 2\mu_2 E'(t).\end{aligned}$$

In other words, for all $t \geq t_0$ we have

$$(\xi(t)G(t) + 2\mu_2 E(t))' \leq \xi'(t)G(t) - \mu_1\xi(t)E(t). \tag{45}$$

Let us to define

$$\mathcal{E}(t) = \xi(t)G(t) + 2\mu_2 E(t). \tag{46}$$

Using the fact that ξ is a positive non-increasing differentiable function, we have $\xi(t) < \xi(0)$ for all $t \geq t_0$. Then, by the lemma 3.3 it is not difficult to see that $\mathcal{E}(t)$ is equivalence to $E(t)$. Therefore, by (21), (45) and (46) we find

$$\mathcal{E}'(t) \leq \alpha_1\xi'(t)E(t) - \mu_1\xi(t)E(t) \leq -\mu_1\xi(t)E(t) \leq -\kappa\xi(t)\mathcal{E}(t), \tag{47}$$

for some positive constant κ. Integrating (47) over (t_0, t), gives the estimate

$$\mathcal{E}(t) \leq \mathcal{E}(0)e^{-\kappa\int_{t_0}^{t}\xi(s)ds}, \qquad \forall t \geq t_0. \tag{48}$$

Consequently, by using (46) and (48), the estimate (26) follows.

4 Exponential Instability

In this section, we will prove that solutions for the problem (1)-(3) grows exponentially. Our main result is summarized in the following theorem.

Theorem 4.1 *Let that the assumptions of Theorem 2.3 be fulfilled. Assume further that $p > m$, $E(0) < 0$ and*

$$\int_0^\infty g(\tau)d\tau < \frac{p-1}{p-1+1/(p+1)}. \tag{49}$$

Then, the L_p norm of any solution u of problem (1)-(3) grows as an exponential function.

Proof. For any $\varepsilon > 0$, let us to define the function

$$\mathcal{L}(t) = H(t) + \varepsilon F(t), \tag{50}$$

where

$$H(t) = -E(t), \tag{51}$$

and

$$F(t) = \int_\Omega u u_t dx + \int_\Omega \nabla u . \nabla u_t dx + \frac{1}{2}\|\nabla u\|_2^2. \tag{52}$$

By (51),(14) and lemma 2.2, since $E(0) < 0$, we find

$$0 < H(0) \leq H(t) \leq \frac{1}{p+1}\|u\|_{p+1}^{p+1}. \tag{53}$$

Differentiate $\mathcal{L}(t)$ with respect to t and using (1) we get

$$\mathcal{L}'(t) = H'(t) + \varepsilon(\|u_t\|_2^2 + \|\nabla u_t\|_2^2 - \|\Delta u\|_2^2)$$
$$+\varepsilon \int_\Omega \int_0^t g(t-\tau)\Delta u(t)\Delta u(\tau) d\tau dx - \varepsilon \int_\Omega u u_t |u_t|^{m-1} dx + \varepsilon \|u\|_{p+1}^{p+1}. \tag{54}$$

Therefore, by (14), the equation (54) can be written in the form

$$\mathcal{L}'(t) = H'(t) + \varepsilon \left(1 + \frac{p+1}{2}\right)(\|u_t\|_2^2 + \|\nabla u_t\|_2^2)$$
$$+\varepsilon \left(\frac{p+1}{2} - 1\right)(1 - \int_0^t g(\tau)d\tau)\|\Delta u\|_2^2 + \varepsilon \left(\frac{p+1}{2}\right)(g \circ \Delta u)(t)$$
$$-\varepsilon \int_\Omega u u_t |u_t|^{m-1} dx - \varepsilon(p+1)E(t)$$
$$+ \varepsilon \int_\Omega \int_0^t g(t-\tau)\Delta u(\Delta u(\tau) - \Delta u(t)) d\tau dx. \tag{55}$$

To estimate the last integral in the right hand side of (55) we use Cauchy-Schwarz inequality and Young's inequality to obtain

$$\int_\Omega \int_0^t g(t-\tau)\Delta u(\Delta u(\tau) - \Delta u(t)) d\tau dx$$

$$\leq \int_0^t g(t-\tau)\|\Delta u(t)\|_2\|\Delta u(\tau)-\Delta u(t)\|_2 d\tau$$

$$\leq \gamma(g\circ\Delta u)(t)+\frac{1}{4\gamma}\int_0^t g(\tau)d\tau\|\Delta u\|_2^2, \qquad \gamma>0. \tag{56}$$

Inserting (56) into (55) we get

$$\mathcal{L}'(t) \geq H'(t)+\varepsilon\left(1+\frac{p+1}{2}\right)(\|u_t\|_2^2+\|\nabla u_t\|_2^2)$$
$$+\varepsilon\left[\left(\frac{p+1}{2}-1\right)-\left(\frac{p+1}{2}-1+\frac{1}{4\gamma}\right)\int_0^t g(\tau)d\tau\right]\|\Delta u\|_2^2$$
$$+\varepsilon\left(\frac{p+1}{2}-\gamma\right)(g\circ\Delta u)(t)-\varepsilon\int_\Omega uu_t|u_t|^{m-1}dx+\varepsilon(p+1)H(t). \tag{57}$$

Choosing $0<\gamma<(p+1)/2$ and using (49), the inequality (57) reduces to

$$\mathcal{L}'(t) \geq H'(t)+\varepsilon\left(1+\frac{p+1}{2}\right)(\|u_t\|_2^2+\|\nabla u_t\|_2^2)+\varepsilon a_1\|\Delta u\|_2^2$$
$$+\varepsilon a_2(g\circ\Delta u)(t)-\varepsilon\int_\Omega uu_t|u_t|^{m-1}dx+\varepsilon(p+1)H(t), \tag{58}$$

where

$$a_1=\left(\frac{p+1}{2}-1\right)-\left(\frac{p+1}{2}-1+\frac{1}{4\gamma}\right)\int_0^\infty g(\tau)\tau,$$
$$a_2=\frac{p+1}{2}-\gamma.$$

For any $\delta>0$, we use the Young's inequality to get

$$\left|\int_\Omega uu_t|u_t|^{m-1}dx\right| \leq \frac{\delta^{m+1}}{m+1}\|u\|_{m+1}^{m+1}+\frac{m}{m+1}\delta^{-(m+1)/m}\|u_t\|_{m+1}^{m+1}. \tag{59}$$

Taking $k=\delta^{-(m+1)/m}$, using (59) and the fact that $H'(t)\geq\|u_t\|_{m+1}^{m+1}$, the estimate (58) takes the form

$$\mathcal{L}'(t) \geq \left(1-\frac{\varepsilon km}{m+1}\right)\|u_t\|_{m+1}^{m+1}+\varepsilon\left(1+\frac{p+1}{2}\right)(\|u_t\|_2^2+\|\nabla u_t\|_2^2)$$

$$+\varepsilon a_1\|\Delta u\|_2^2+\varepsilon a_2(g\circ\Delta u)(t)-\varepsilon\frac{k^{-m}}{m+1}\|u\|_{m+1}^{m+1}+\varepsilon(p+1)H(t). \tag{60}$$

Since $p>m$, by the imbedding $L^{p+1}(\Omega)\hookrightarrow L^{m+1}(\Omega)$ we obtain

$$\|u\|_{m+1}^{m+1}\le C\|u\|_{p+1}^{m+1}, \tag{61}$$

for some positive constant C. Using the algebraic inequality

$$z^\nu\le(1+z)\le\left(1+\frac{1}{a}\right)(z+a),\qquad \forall z\ge 0,\quad 0<\nu\le 1,\quad a\ge 0,$$

we find

$$\left(\|u\|_{p+1}^{p+1}\right)^{\frac{m+1}{p+1}}\le d\left(\|u\|_{p+1}^{p+1}+H(0)\right), \tag{62}$$

where $d=1+1/H(0)$. Considering the inequalities (53), (61), (62) and inserting the result into (60) we obtain

$$\begin{aligned}\mathcal{L}'(t)\ge&\left(1-\frac{\varepsilon km}{m+1}\right)\|u_t\|_{m+1}^{m+1}+\varepsilon\left(1+\frac{p+1}{2}\right)(\|u_t\|_2^2+\|\nabla u_t\|_2^2)\\&+\varepsilon a_1\|\Delta u\|_2^2+\varepsilon a_2(g\circ\Delta u)(t)-\varepsilon\frac{k^{-m}}{m+1}Cd\left(1+\frac{1}{p+1}\right)\|u\|_{p+1}^{p+1}\\&+\varepsilon(p+1)H(t).\end{aligned} \tag{63}$$

For the last term in the right hand side of (63), from the definition of $H(t)$ we have

$$H(t)\ge-\frac{1}{2}\left[\|u_t\|_2^2+\|\nabla u_t\|_2^2+\|\Delta u\|_2^2+(g\circ\Delta u)(t)\right]+\frac{1}{p+1}\|u\|_{p+1}^{p+1}. \tag{64}$$

Consequently, choosing $a_3<\min\{(p+1)/2,a_1,a_2\}$ and using (64), the estimate (63) can be rewritten in the form

$$\begin{aligned}\mathcal{L}'(t)\ge&\left(1-\frac{\varepsilon km}{m+1}\right)\|u_t\|_{m+1}^{m+1}+\varepsilon\left(1+\frac{p+1}{2}-a_3\right)(\|u_t\|_2^2+\|\nabla u_t\|_2^2)\\&+\varepsilon(a_1-a_3)\|\Delta u\|_2^2+\varepsilon(a_2-a_3)(g\circ\Delta u)(t)\\&+\varepsilon\left[\frac{2a_3}{p+1}-\frac{k^{-m}}{m+1}Cd\left(1+\frac{1}{p+1}\right)\right]\|u\|_{p+1}^{p+1}+\varepsilon(p+1-2a_3)H(t).\end{aligned}$$

Now, we select k so large such that

$$\frac{2a_3}{p+1} - \frac{k^{-m}}{m+1} Cd \left(1 + \frac{1}{p+1}\right) > 0.$$

Whence k is fixed, we choose ε so small enough such that $\varepsilon < \frac{m+1}{km}$. Therefore, there exists $\sigma_1 > 0$ so that

$$\mathcal{L}'(t) \geq \varepsilon\sigma_1 \left(\|u_t\|_2^2 + \|\nabla u_t\|_2^2 + \|\Delta u\|_2^2 + (g \circ \Delta u)(t) + \|u\|_{p+1}^{p+1} + H(t)\right). \tag{65}$$

Hence

$$\mathcal{L}(t) \geq \mathcal{L}(0) > 0, \qquad \forall t \geq 0,$$

where

$$\mathcal{L}(0) = H(0) + \varepsilon \left[\int_\Omega u_0 u_1 dx + \int_\Omega \nabla u_0 . \nabla u_1 dx + \frac{1}{2}\|\nabla u_0\|_2^2\right].$$

On the other hand, Young's inequality and Poincaré inequality imply that

$$\int_\Omega u u_t dx \leq \frac{1}{2}\|u\|_2^2 + \frac{1}{2}\|u_t\|_2^2 \leq \frac{\lambda^{-2}}{2}\|\Delta u\|_2^2 + \frac{1}{2}\|u_t\|_2^2, \tag{66}$$

$$\int_\Omega \nabla u . \nabla u_t dx \leq \frac{1}{2}\|\nabla u\|_2^2 + \frac{1}{2}\|\nabla u_t\|_2^2 \leq \frac{\lambda^{-1}}{2}\|\Delta u\|_2^2 + \frac{1}{2}\|\nabla u_t\|_2^2, \tag{67}$$

where λ is the Poincaré constant. Therefore, using (50), (52) and the estimates (66) and (67) we have

$$\mathcal{L}(t) \leq \sigma_2 \left(H(t) + \|u_t\|_2^2 + \|\nabla u_t\|_2^2 + \|\Delta u\|_2^2 + (g \circ \Delta u)(t) + \|u\|_{p+1}^{p+1}\right), \tag{68}$$

where σ_2 is a positive constant. By the estimates (65) and (68) we deduce

$$\mathcal{L}'(t) \geq \varepsilon \frac{\sigma_1}{\sigma_2} \mathcal{L}(t).$$

Therefore,

$$\mathcal{L}(t) \geq \mathcal{L}(0) \exp\left(\varepsilon \frac{\sigma_1}{\sigma_2} t\right). \tag{69}$$

Considering the definition of $\mathcal{L}(t)$ we have $\mathcal{L}(t) \leq 1/(p+1)\|u\|_{p+1}^{p+1}$ (for sufficient small $\varepsilon > 0$) and hence the desired result can be obtained from (69).

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