A New Method for Improving the Discrimination Power and Weights Dispersion in the Data Envelopment Analysis

S. Kordrostami
Islamic Azad University, Lahijan Branch

A. Mirmousavi[*]
Islamic Azad University, Lahijan Branch

Abstract. The appropriate choice of input-output weights is necessary to have a successful DEA model. Generally, if the number of DMUs i.e., n, is less than number of inputs and outputs i.e., m+s, then many of DMUs are introduced as efficient then the discrimination between DMUs is not possible. Besides, DEA models are free to choose the best weights. For resolving the problems that are resulted from freedom of weights, some constraints are set on the input-output weights. Symmetric weight constraints are a kind of weight constrains. In this paper, we represent a new model based on a multi-criterion data envelopment analysis (MCDEA) are developed to moderate the homogeneity of weights distribution by using symmetric weight constrains.Consequently, we show that the improvement of the dispersal of unrealistic input-output weights and the increasing discrimination power for our suggested models. Finally, as an application of the new model, we use this model to evaluate and ranking guilan selected hospitals.

1.   Introduction

Data Envelopment Analysis (DEA) was first developed by Charnes et al. [4] to measure the relative efficiencies of a set of decision making units (DMUs) with multiple inputs and multiple outputs. Beside of its popularity, DEA has some drawbacks such as unrealistic input-output weights and lack of discrimination among efficient DMUs. In DEA we sometimes encounter extreme values or zero in input and/or output weights for examined DMUs. In some cases we meet the unfitness of weights, i.e., a solution giving a big weight to variables with less importance or giving a small or zero weight to important variables. Especially in the zero cases, weights of input and/or output do not contribute to interpret the results of analysis. The problem of unrealistic weights in DEA has tackled mainly by the techniques weight restriction. In the literature, various efforts have been done to overcome this problem [1, 6, 9]. For resolving the problems that are resulted from freedom of weights, some constraints are set on the input-output weights. Symmetric weight constraints are a kind of weight constrains that proposed by Dimitrov and Sutton [5]. Li and Reeves [7] have been developed multi-criteria data envelopment analysis (MCDEA) with the aid of one criterion efficiency evaluation methods. The Li and Reeves approach gives non-dominant (non-optimal) solutions. These non-dominant solutions can also be different to the preferences of decision maker. Bal et al. [2] proposed a CVDEA model which incorporates the coefficient of variation (CV) for input and output weights reducing the number of efficient DMUs and produces more homogeneous weight dispersion for input-outputs. However, the CVDEA model should be used providently since it does not preserve the unit-invariance and linearity properties. In this paper, new and easy-to-use model are presented to overcome this problem and generally more balanced input-output weights dispersions are obtained with respect to the basic DEA models and also reduced the number of efficient DMUs with additional symmetric weight constraints on weights. In order to, input and output weights values of DMU under evaluation become nonzero and symmetry. In addition, show the improvement of the dispersion of unrealistic input-output weights and the increasing

discrimination power for our suggested models. Also, suggested models have unit-invariance property. For our new approach it is not necessary any a priori information involving human value judgment, as well, more-over it seems there is no unfeasibility problem for the solutions to our approach.

The study is organized as follows. In section 2, the CCR and the multi-criteria data envelopment analysis (MCDEA) models are briefly explained. In section 3, we present the suggest model. In section 4, the performances of the CCR and the suggested models are compared by a real word data. In section 5, a summary of the research and its results are provided.

2. Backgrounds

2.1 Data Envelopment Analysis

Data envelopment analysis (DEA) is a fractional mathematical program-ming technique that has been developed by Charnes et al. [4]. It is used to measure the relative efficiencies of decision making units (DMUs) that use similar inputs to product similar outputs where the multiple inputs and outputs are incommensurate in nature. DEA has been one of the fastest growing areas of Operations Research and Management Science in the past. Also, the ability to estimate too strongly to analyses the efficiency of state and private organizations, link: Schools, Hospitals, Banks, Stores, and etc. Suppose there are n DMUs to be evaluated in terms of m inputs and s outputs. Let $x_{ij}(i = 1, 2, \ldots, m)$ and $y_{rj}(r = 1, 2, \ldots, s)$ be the input and output values of $DMU_j(j = 1, 2, \ldots, n)$. The relative efficiency of a particular is obtained by solving the follow-ing fractional programming problem [4]:

$$
\begin{aligned}
w_o \;=\; & Max \; \left(\sum_{r=1}^{s} u_r y_{ro}\right) / \left(\sum_{i=1}^{m} v_i x_{io}\right) \\
s.t. \; & \left(\sum_{r=1}^{s} u_r y_{rj}\right) / \left(\sum_{i=1}^{m} v_i x_{ij}\right) \;\leqslant\; 1, \; j = 1, 2, \ldots, n, \quad\quad (1) \\
& u_r \;\geqslant\; 0, \; r = 1, 2, \ldots, s, \\
& v_i \;\geqslant\; 0, \; i = 1, 2, \ldots, m.
\end{aligned}
$$

Where j is the DMU index, $j = 1, 2, \ldots, n$; r the output index, $r = 1, 2, \ldots, s$; i is the input index, $i = 1, 2, \ldots, m$; y_{rj} the value of the rth output for jth DMU, x_{ij} the value of the ith input for jth DMU, u_r the weight given to the rth output; v_i the weight given to the ith input, and w_o is the relative efficiency of DMU_o, the DMU under evaluation or the target DMU. In this model, DMU_o is efficient if and only if $w_o = 1$, or else, DMU_o is inefficient if and only if $w_o < 1$. This fractional programming model can be converted into a linear programming problem where the optimal value of the objective function indicates the relative efficiency of DMU_o. The reformulated linear programming problem, also known as the CCR model, is as follows:

$$w_o = Max \left(\sum_{r=1}^{s} u_r y_{ro} \right)$$

$$s.t. \left(\sum_{i=1}^{m} v_i x_{io} \right) = 1$$

$$\left(\sum_{r=1}^{s} u_r y_{rj} \right) - \left(\sum_{i=1}^{m} v_i x_{ij} \right) \leqslant 1, \quad j = 1, 2, \ldots, n, \tag{2}$$

$$u_r \geqslant 0, \quad r = 1, 2, \ldots, s,$$

$$v_i \geqslant 0, \quad i = 1, 2, \ldots, m.$$

In model (2), i.e., the classical DEA, the weighted sum of the input for the target DMU is forced to 1, thus allowing for the conversion of the fractional programming problem into a linear programming problem that can be solved by using any linear programming software. The solution to model (2) assigns the value 1 to all efficient DMUs. The super efficiency concept is proposed to differentiate completely among all efficient DMUs when there are more than one efficient DMUs. One of the super efficiency models for ranking efficient DMUs in DEA was introduced by Andersen and Petersen [1]. This method enables an extreme efficient unit Oth to achieve an efficiency score greater than one by removing the Oth constraint in the envelopment LP formulation, as

shown in model (3).

$$(AP) \quad Max \ (\sum_{r=1}^{s} u_r y_{ro})$$

$$s.t. \ (\sum_{i=1}^{m} v_i x_{io}) \ = \ 1$$

$$(\sum_{r=1}^{s} u_r y_{rj}) - (\sum_{i=1}^{m} v_i x_{ij}) \ \leqslant \ 1, \quad j = 1, 2, \dots, n, j \neq o, \qquad (3)$$

$$u_r \ \geqslant \ 0, \quad r = 1, 2, \dots, s,$$

$$v_i \ \geqslant \ 0, \quad i = 1, 2, \dots, m.$$

2.2 Multiple Criteria DEA Model

Consider the following problem:

$$Max \ (f_1(x), f_2(x), f_3(x), \dots, f_K(x))$$

$$s.t. \ x \in X, \qquad (4)$$

Where $f_1, f_2, , f_k$ are objective functions and X is non-empty feasible region. The model (4) is called multiple objectives Linear programming (MOLP). Generally, model CCR, (2), can be expressed equivalently in the form given as follows [7]:

$$Min \ d_o \ (or \ Max \ (\sum_{r=1}^{s} u_r y_{ro}))$$

$$s.t. \ (\sum_{i=1}^{m} v_i x_{io}) \ = \ 1, \qquad (5)$$

$$(\sum_{r=1}^{s} u_r y_{rj}) - (\sum_{i=1}^{m} v_i x_{ij}) + d_j = 0, j = 1, 2, \dots, n$$

$$u_r \ \geqslant \ 0, \quad r = 1, 2, \dots, s,$$

$$v_i \ \geqslant \ 0, \quad i = 1, 2, \dots, m$$

$$d_j \ \geqslant \ 0, \quad j = 1, 2, \dots, n.$$

Where d_o is the deviation variable for DMU_o and d_j is the deviation variable of DMU_j. The quantity d_o, which is bounded by the interval [0, 1], can be regarded as a measure of inefficiency. A multiple criteria data envelopment analysis model formulation with the minmax and minsum criteria, which minimizes a deviation variable, d_o, rather than maximizing the efficiency score, $max \sum_{r=1}^{s} u_r y_{ro}$. Hence, MCDEA model is shown below [7]:

$$Min \ d_o \ (Max \ (\sum_{r=1}^{s} u_r y_{ro}))$$

$$or \ Min \ M \ (where \ M \ = \ max \ d_j)$$

$$or \ Min \ \sum_{j=1}^{n} d_j$$

$$s.t. \ (\sum_{i=1}^{m} v_i x_{io}) \ = \ 1, \tag{6}$$

$$(\sum_{r=1}^{s} u_r y_{rj}) - (\sum_{i=1}^{m} v_i x_{ij}) + d_j \ = \ 0, \quad j = 1, 2, \ldots, n,$$

$$M - d_j \geqslant 0 \ , j = 1, 2, \ldots, n,$$

$$u_r \ \geqslant \ 0, \quad r = 1, 2, \ldots, s,$$

$$v_i \ \geqslant \ 0, \quad i = 1, 2, \ldots, m$$

$$d_j \ \geqslant \ 0, \quad j = 1, 2, \ldots, n.$$

Note, the larger the deviation variable, which is also bounded by interval [0, 1], the lesser efficient is DMU_o. The first objective function, $min \ d_o$, is the classical DEA objective. The second objective function, min M, is a minmax function minimizing the maximum deviation variable. The third objective function, $Min \ \sum_{j=0}^{n} d_j$, is a minsum function minimizing the sum of the deviation variables. The constraints $M - d_j \geqslant 0, j = 1, 2, \ldots, n$ that define the maximum deviation M do not change the feasible region of decision variables [7]. Model (6) is an MOLP model. In an MOLP problem, it is generally impossible to find a solution that optimizes all objective simultaneously. Therefore, the task of an MOLP solution process is not to find an optimal solution

but, instead, to find non-dominated solutions and to help select a most preferred one. Loosely speaking, a solution, represented by a point in decision variable space, is non-dominated if it is not possible to move the point within the feasible region to improve an objective function value without deteriorating at least one of the other objectives. In multiple criteria terminology, a non-dominated solution is also called an efficient solution. To prevent possible confusion with DEA,s efficiency concept, we will use the term "non-dominated". For more details about multiple objective optimizations, readers are referred to Steuer [12] and Cohon [11]. One fact we would like to point out here is that a non-dominated solution set for an MOLP problem will always contain, but is not limited to, the optimal solutions obtained by individually optimizing each of the objectives in the MOLP problem under the setting of single objective linear programming (LP). Now, it is quite obvious that the solution that optimizes the first objective function of model (6) is equivalent to the optimal solution of model (2) or (5). That is, DMU_o is efficient (in the classical sense) if and only if the value of d_o corresponding to the solution that optimizes the first objective function of model (6) is zero. Based on the same research spirit, we can define a DMUs relative efficiency corresponding to the second and third criteria in following way: DMU_o is minimax efficient if and only if the value of d_o corresponding to the solution that minimizes the second objective function of model (6) is zero; similarly, DMU_o is minsum efficient if and only if the value of d_o corresponding to the solution that minimizes the third objective function of model (6) is zero. In all three above definitions, no matter if DMU_o is efficient or not, its DEA efficiency score is $1 - d_o$, but the values of d_o can very under different criteria. The minimax and minsum criteria do not give favorable consideration to the DMU under evaluation, as the classical DEA criterion does. Therefore, efficiencies defined under minimax and minsum criteria are more restrictive than that defined in classical DEA; that is, it is more difficult for a DMU to achieve minimax or minsum efficiency than to achieve classical DEA efficiency. More precisely, if DMU_o is minimax or minsum efficient, it must also be DEA efficient, because, by definition, minimax or minsum efficiency requires $d_o = 0$. However, if DMU_o is DEA efficient, it may or may not be

minimax or minsum efficient, because $d_o = 0$ does not necessarily imply that $M \, or \, \sum_{j=1}^{n} d_j$ is minimized. Based on this fact, we can conclude that the minimax or minsum criterion generally yields fewer efficient DMUs. Including these new criteria in a DEA model, therefore, will result in the improvement in discriminating power. On the other hand, since $M \, and \, \sum_{j=1}^{n} d_j$ are functions of all deviation variables and each deviation variable is related to a constraint, minimizing M or $\sum_{j=1}^{n} d_j$ is, in some sense, equivalent to imposing tighter constraints on weight variables. In this way, weight flexibility is effectively restricted. Model (6) is only one form of a MCDEA model. The selection of efficiency criteria depends on the purpose of a study. Often, the minimax criterion is more restrictive than the minsum criterion. Both criteria tend to yield too few efficient DMUs when the number of DMUs is large compared to the total number of inputs and outputs and/or when the interest of studies is to form an efficient frontier. In this case, other efficiency criteria, such as the weighted sum of all deviation variables in which more favorable weight is assigned to the DMU under evaluation, can be employed. On the other hand, if the minimax criterion is still not tight enough in terms of discriminating power, more restrictive criterion can be adopted.

2.3   Definitions

The lack of discrimination power problem occurs when the number of DMUs under evaluation is not large enough compared to the total number of inputs-outputs. In this situation, classical DEA models often yield solutions that identify too many DMUs as efficient. To better the lack discrimination power of DEA, some DEA approaches; such as super efficiency, multiple criteria DEA and cross efficiency, have been proposed in DEA literature. However, in some cases it is also possible to meet the infeasibility problems in super efficiency models and complexity of multiple objectives for multiple criteria DEA models. The cross efficiency approach is a useful technique developed by sexton et al. [10]. Although the cross efficiency method has a widespread usage, it has also some deficiencies arising from the classical DEA [2,3,7]. The problem of unrealistic

weight dispersion for DEA occurs when some DMUs are rated as efficient because of input and output weights have the extreme or zero values. To overcome the unrealistic weight dispersion problem, weight restriction techniques such as cone ratio envelopment, assurance region and value efficiency have been proposed in DEA literature. However, these techniques are depended on the measurement units of inputs-outputs and may also give infeasible solutions for weights [2,3,7]. However, this problem can be resolved by use symmetric weight constraints for input-output weights which expect input-output weights become nonzero and symmetry [5].

3.   Suggest Model

Lack of discrimination power in DEA basic methods can be resolved by a single objective function, instead of factors and very chaotic function. Therefore, considering model (6), we consider the below model as main base for presenting the suggested model.

$$
Max \ (\sum_{r=1}^{s} u_r y_{ro})
$$
$$
s.t. \ (\sum_{i=1}^{m} v_i x_{io}) \ = \ 1, \tag{7}
$$
$$
(\sum_{r=1}^{s} u_r y_{rj}) - (\sum_{i=1}^{m} v_i x_{ij}) + d_j \ = \ 0, \ \ j = 1, 2, \ldots, n,
$$
$$
M - d_j \geqslant 0, \ \ j = 1, 2, \ldots, n,
$$
$$
u_r \ \geqslant \ 0, \ \ r = 1, 2, \ldots, s,
$$
$$
v_i \ \geqslant \ 0, \ \ i = 1, 2, \ldots, m
$$
$$
d_j \ \geqslant \ 0, \ \ j = 1, 2, \ldots, n.
$$

In the process of improving discrimination power by model (7), some of input-output weights will be zero, and this means, there is problem about weights dispersion. This can be resolved by adding below sym-

metric weight constraints to model (8).

$$|u_p y_{po} - u_q y_{qo}| = z_{pq} \quad ,p,q = 1,2,\ldots,s, \qquad (8)$$
$$|v_E x_{Eo} - v_F x_{Fo}| = w_{EF} \quad ,E,F = 1,2,\ldots,m.$$

For making simultaneous symmetry of input-output weights, we suggest that all amounts in z_{pq} and w_{EF} must be maximum in objective function. z_{pq} in (8) is the difference in symmetry between output dimension p and dimension q for the DMU under evaluation. w_{EF} in (8) is the difference in symmetry between input dimension E and dimension F for the DMU under evaluation. Besides, we evaluate the efficiency of symmetry by a symmetry scaling factor, $\beta \geqslant 0$, [5]. Then, the new model is showed as below:

$$Max \; (\sum_{r=1}^{s} u_r y_{ro}) - \beta(\sum_{p=1}^{s}\sum_{q=1}^{s} z_{pq} + \sum_{E=1}^{m}\sum_{F=1}^{m} w_{EF})$$

$$s.t. \; (\sum_{i=1}^{m} v_i x_{io}) = 1, \qquad (9)$$

$$(\sum_{r=1}^{s} u_r y_{rj}) - (\sum_{i=1}^{m} v_i x_{ij}) + d_j = 0, \quad j = 1,2,\ldots,n,$$

$$M - d_j \leqslant 0, \quad j = 1,2,\ldots,n,$$

$$|u_p y_{po} - u_q y_{qo}| = z_{pq} \quad ,p,q = 1,2,\ldots,s,$$
$$|v_E x_{Eo} - v_F x_{Fo}| = w_{EF} \quad ,E,F = 1,2,\ldots,m$$
$$u_r, u_p, u_q \geqslant 0, \quad r = 1,2,\ldots,s,$$
$$v_i, v_E, v_F \geqslant 0, \quad i = 1,2,\ldots,m$$
$$d_j \geqslant 0, \quad j = 1,2,\ldots,n.$$

Note that (9) is not linear with the equality constraint. Rewrite (9) to a linear program as:

$$Max \ (\sum_{r=1}^{s} u_r y_{ro}) - \beta(\sum_{p=1}^{s}\sum_{q=1}^{s} z_{pq} + \sum_{E=1}^{m}\sum_{F=1}^{m} w_{EF}$$

$$s.t. \ (\sum_{i=1}^{m} v_i x_{io}) \ = \ 1, \quad (10)$$

$$(\sum_{r=1}^{s} u_r y_{rj}) - (\sum_{i=1}^{m} v_i x_{ij}) + d_j \ = \ 0, \quad j = 1, 2, \ldots, n,$$

$$M - d_j \leqslant 0, j = 1, 2, \ldots, n,$$

$$u_p y_{po} - u_q y_{qo} \leqslant z_{pq} \quad, p, q = 1, 2, \ldots, s,$$

$$u_p y_{po} + u_q y_{qo} \leqslant z_{pq} \quad, p, q = 1, 2, \ldots, s,$$

$$v_E x_{Eo} - v_F x_{Fo} \leqslant w_{EF} \quad, E, F = 1, 2, \ldots, m,$$

$$v_E x_{Eo} + v_F x_{Fo} \leqslant w_{EF} \quad, E, F = 1, 2, \ldots, m,$$

$$u_r, u_p, u_q \geqslant 0 \quad, r, p, q = 1, 2, \ldots, s,$$

$$v_i, v_E, v_F \geqslant 0 \quad, i, E, F = 1, 2, \ldots, m,$$

$$d_j \geqslant 0 \quad, j = 1, 2, \ldots, n.$$

Note that the linear program (10) has the same feasibility region as the linear program (2) and (7) [5].

Comments The above suggested model has the following characteristics:
1. It can be easily discriminated between efficient and inefficient units.
2. Input-Output weights of under evaluation units will be zero by decreasing β.
3. By decreasing β, under evaluation units achieve simultaneous symmetry in input-output weights.
4. Objective function is also bounded by interval [0, 1].
5. Envelopment form of above model is also unit-invariance property.

4. Numerical Examples

Example 4.1. (Efficiency evaluation of seven departments in a univer-

sity, Wong and Beasley [8]). Seven departments (DMUs) in a university
are evaluated in terms of three inputs and three outputs give below and
their related input and output data are provided in table 1.

x_1 : Number of academic staff.

x_2 : Academic staff salaries in thousands of pounds.

x_3: Support staff salaries in thousands of pounds.

y_1 : Number of undergraduate students.

y_2: Number of postgraduate students.

y_3 : Number of research papers.

The results of models for the Wong and Beasley data are summarized
in Table 2 and 3. In Table 2, the basic CCR model finds 6 of 7 DMUs
as efficient. Also, some of input-output weights will be zero, and this
means, there existent problem about weights dispersion. In order to, the
zero weights are not taken into account to interpret the efficiency of the
DMUs. In Table 3, for the suggested model (10) input-output weights
of under evaluation units will not be zero.

Table 1: Data for seven departments in a university.

DMU	y_1	y_2	y_3	x_1	x_2	x_3
1	60	35	17	12	400	20
2	139	41	40	19	750	70
3	225	68	75	42	1500	70
4	90	12	17	15	600	100
5	253	145	130	45	2000	250
6	132	45	45	19	730	50
7	305	159	97	41	2350	600

Table 2: The results of the basic CCR model for the university data set.

DMU	EFF	AP	RANK	U_1	U_2	U_3	V_1	V_2	V_3
1	1.000	1.829	1	0.000	0.286	0.000	0.795	0.000	0.022
2	1.000	1.048	6	0.056	0.053	0.000	0.493	0.000	0.009
3	1.000	1.193	4	0.444	0.000	0.000	0.116	0.000	0.073
4	0.820	0.820	7	0.091	0.000	0.000	0.641	0.001	0.000
5	1.000	1.217	3	0.016	0.000	0.045	0.204	0.000	0.000
6	1.000	1.190	5	0.038	0.000	0.109	0.488	0.001	0.000
7	1.000	1.254	2	0.017	0.000	0.049	0.218	0.000	0.000

Table 3: The results of the suggested model (10) for the university data set $(\beta = 1)$.

DMU	EFF	RANK	U_1	U_2	U_3	V_1	V_2	V_3
1	0.963	2	0.054	0.092	0.188	0.277	0.008	0.166
x 2	0.847	3	0.020	0.068	0.071	0.175	0.004	0.048
3	0.831	5	0.012	0.041	0.036	0.079	0.002	0.048
4	0.363	7	0.013	0.101	0.071	0.222	0.005	0.033
5	0.838	4	0.011	0.019	0.021	0.074	0.002	0.013
6	1.000	1	0.025	0.073	0.074	0.175	0.004	0.066
7	0.727	6	0.008	0.015	0.025	0.081	0.001	0.055

The results obtained in Table 2 and 3, show that the dispersion (variation) of input-output weights assigned to DMUs by the suggested model (3.4) are less than, i.e., more homogeneous than those of basic DEA models. In addition, there is a powerful correlation in the same direction between efficiency ranking values of the DMUs obtained by CCR and suggested model (3.4). In order to, the suggested model yielded a more balanced dispersion of input-output weights and reduced the number of efficient DMUs.

Example 4.2. (Efficiency evaluation of guilan selected hospitals, collected in JUN 2010). There are different methods for evaluation of the efficiency of hospitals, DEA technique is one of the best methods, because of use of mathematics technique and avoiding of mental methods. Our suggested model for efficiency evaluation of hospitals is a model based on symmetric weight constraints. In fact, in this model, also yield more homogenous dispersion for input-output weights. For efficiency evaluation of guilan hospitals, we use a real word data of nine selected hospitals. Nine hospitals (DMUs) are evaluated in terms of five inputs and three outputs give below and their related input and output data are provided in table 4.

x_1 : The number of general practitioners in each hospital.

x_2: The number of specialist physicians in each hospital.

x_3 : The number of personnel of nursing in each hospital.

x_4 : The cost of each hospital, including the personnels salary, the cost of buying the medication, the medical equipments, and etc (in 10 thou-

sands dollars).

x_5 : The number of active sickbed in each hospital.

y_1 : The number of hospitalized patients, regarding to monthly reception.

y_2 : The number of walking patients in each hospital.

y_3 : The amount of the income in each hospital that is including cash and non-cash incomes (in 10 thousands dollars). The results of models for the hospital data are summarized in Table 5 and 6. In Table 5, the basic CCR model finds 8 of 9 DMUs as efficient. In Table 6, for the suggested model (10) input-output weights of under evaluation units will not be zero.

Table 4: Data for nine hospitals (collected in JUN 2010).

DMU	X_1	X_2	X_3	X_4	X_5	Y_1	Y_2	Y_3
1	17	76	553	113.9	272	3587	67275	120
2	12	24	219	70	136	1728	4217	76.7
3	8	52	342	32	81	1708	9563	85
4	2	22	84	39.8	105	463	5522	21.6
5	14	39	145	18.8	167	652	8583	20.1
6	13	46	167	29	140	1130	3859	30
7	13	33	150	19.2	150	1318	3900	20.5
8	8	18	184	98	154	812	2655	10.3
9	14	17	99	127	99	863	304	14.9

Table 5: The results of the basic CCR model for the hospital data set.

DMU	EFF	AP	RANK	U_1	U_2	U_3	V_1	V_2	V_3	V_4	V_5
1	1.000	1.239	6	0.061	0.000	0.000	0.280	0.000	0.000	0.000	0.019
2	1.000	1.936	2	0.103	0.000	0.001	0.468	0.000	0.000	0.000	0.032
3	1.000	1.262	4	0.095	0.000	0.001	0.433	0.000	0.000	0.000	0.029
4	1.000	2.303	1	0.000	0.001	0.006	0.464	0.000	0.000	0.000	0.086
5	1.000	1.711	3	0.000	0.001	0.004	0.307	0.000	0.000	0.000	0.057
6	0.941	0.941	9	0.279	0.001	0.001	0.064	0.000	0.000	0.316	0.000
7	1.000	1.173	7	0.401	0.001	0.001	0.092	0.000	0.001	0.454	0.000
8	1.000	1.242	5	0.764	0.000	0.002	0.176	0.000	0.001	0.865	0.000
9	1.000	1.109	8	0.579	0.000	0.001	0.133	0.000	0.001	0.655	0.000

Table 6: The results of the suggested model (10) for the hospital data set ($\beta = 1$).

DMU	EFF	RANK	U_1	U_2	U_3	V_1	V_2	V_3	V_4	V_5
1	0.596	8	0.016	0.0003	0.0005	0.117	0.026	0.004	0.017	0.007
2	0.814	3	0.035	0.0006	0.0015	0.116	0.083	0.009	0.028	0.015
3	0.909	2	0.036	0.0003	0.0017	0.250	0.038	0.006	0.025	0.009
4	1.000	1	0.154	0.0006	0.0072	1.000	0.090	0.025	0.096	0.019
5	0.696	6	0.115	0.0002	0.0035	0.143	0.051	0.014	0.106	0.020
6	0.605	7	0.067	0.0005	0.0017	0.154	0.043	0.012	0.069	0.014
7	0.706	5	0.115	0.0006	0.0017	0.154	0.061	0.013	0.104	0.016
8	0.721	4	0.233	0.0009	0.0029	0.250	0.111	0.017	0.204	0.026
9	0.151	9	0.034	0.0016	0.0005	0.143	0.176	0.020	0.163	0.026

The results obtained in Table 5 and 6, show that the dispersion (variation) of input-output weights assigned to DMUs by the suggested model (10) are less than, i.e., more homogeneous than those of basic DEA models. In addition, there is a powerful correlation in the same direction between efficiency ranking values of the DMUs obtained by CCR and suggested model (10). In order to, the suggested model yielded a more balanced dispersion of input-output weights and reduced the number of efficient DMUs.

5. Conclusions

In DEA we sometimes encounter extreme values or zero in input and/or output weights of DMUs under evaluation. In some cases we meet the unfitness of weights, i.e., a solution giving a big weight to variables with less importance or giving a small or zero weight to important variables. This paper overcomes this problem by new model based on a Multi-Criterion Data Envelopment Analysis (MCDEA) are developed to moderate the homogeneity of weights distribution by using symmetric weight constrains. Also, the suggested model is applied to the two instances whit a real-word data, it has been seen that the suggested model improves the dispersal of unrealistic input-output weights and the increasing discrimination power. Consequently, the suggested model yielded a more balanced dispersion of input-output weights and reduces the number of efficient DMUs.

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Sohrab Kordrostami
Department of Mathematics
Associate Professor of Mathematics
Islamic Azad University, Lahijan Branch
Lahijan, Iran
E-mail: sohrabkordrostami@gmail.com

Ali Mirmousavi
Department of Mathematics
Ph.D student of Mathematics
Islamic Azad University, Lahijan Branch
Lahijan, Iran
E-mail: mosavi52@yahoo.com