Journal of Mathematical Extension
Vol. 7, No. 3, (2013), 95-106

Generalizations of ϵ-Fixed Point Theorems in Partial Metric Spaces

S. A. M. Mohsenalhosseini*
Vali-e-Asr University

H. Mazaheri
Yazd University

Abstract. We consider the dualistic partial metric spaces on a set X, and we give necessary conditions for existence of fixed point and $\epsilon-$fixed point for some maps.

AMS Subject Classification: 54H25; 54E50; 54E99; 68Q55
Keywords and Phrases: Fixed points, $\epsilon-$ fixed point, partial metric spaces, dualistic partial metric spaces

1. Introduction

The partial metric spaces has been introduced by Matthews in [5] as a part of the study of denotational semantics of dataflow networks. In particular, Matthews established the precise relationship between partial metric spaces and the so-called weightable quasi-metric spaces. Indeed he proved a partial metric generalization of Banach contraction mapping theorem.

A partial metric [5] on a set X is a function $p : X \times X \to [0, \infty)$ such that for all $x, y, z \in X$:

(1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$;

(2) $p(x, x) \leqslant p(x, y)$;

Received: April 2013; Accepted: July 2013
*Corresponding author

(3) $p(x,y) = p(y,x)$;
(4) $p(x,z) \leqslant p(x,y) + p(y,z) - p(y,y)$.
A partial metric space is a pair (X,p), where p is a partial metric on X. If p is a partial metric on X, then the function $p^s : X \times X \to [0,\infty)$ given by $p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y)$ is a (usual) metric on X. Each partial metric p on X induces a T_0 topology τ_p on X which has as a basis of the family of open p-balls $\{B_p(x,\epsilon) : x \in X, \epsilon > 0\}$, where $B_p(x,\epsilon) = \{y \in X : \ p(x,y) < p(x,x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$. Similarly, closed p-ball is defined as $B_p(x,\epsilon) = \{y \in X : \ p(x,y) \leqslant p(x,x) + \epsilon\}$.
A sequence $\{x_n\}_{n\in N}$ in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite) $lim_{n,m} p(x_n,x_m)$ [5].
A partial metric space (X,p) is said to be complete if every Cauchy sequence $\{x_n\}_{n\in N}$ in X converges, with respect to τ_p to a point $x \in X$ such that $p(x,x) = lim_{n,m} p(x_n,x_m)$ [5].
A mapping $T : X \to X$ is said to be continuous at $x_0 \in X$, if for $\epsilon > 0$, there exists $\delta > 0$ such that $T(B_p(x_0,\delta)) \subset B_p(T(x_0),\epsilon)$. [1]

Definition 1.1. [5] *An open ball for a partial metric $p : X \times X \to [0,\infty)$ is a set of the form $B^p_\epsilon(x) ::= \{y \in X : \ p(x,y) < \epsilon\}$ for each $\epsilon > 0$ and $x \in X$.*
In [9], S. J. O'Neill proposed one significant change to Matthews definition of the partial metrics, and that was to extend their range from $R^+$to R. In the following, partial metrics in the O'Neill sense will be called dualistic partial metrics and a pair (X,p) such that X is a nonempty set and p is a dualistic partial metric on X will be called a dualistic partial metric space.
A dualistic partial metric on a set X is a function $p : X \times X \to \mathbb{R}$ such that for all $x,y,z \in X$:
(1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$;
(2) $p(x,x) \leqslant p(x,y)$;
(3) $p(x,y) = p(y,x)$;
(4) $p(x,z) \leqslant p(x,y) + p(y,z) - p(y,y)$. A dualistic partial metric space is a pair (X,p), where p is a dualistic partial metric on X.
A quasi-metric on a set X we mean a nonnegative real-valued function d on $X \times X$ such that for all $x,y,z \in X$:

(i) $d(x, y) = d(y, x) = 0 \Leftrightarrow x = y$,
(ii) $d(x, y) \leqslant d(x, z) + d(z, y)$.
A quasi-metric space is a pair (X, d) *such that* X *is a (nonempty) set and* d *is a quasi-metric on* X.

Lemma 1.2. [5] *If* (X, p) *is a dualistic partial metric space, then the function* $d_p : X \times X \to R^+$ *defined by* $d_p(x, y) = p(x, y) - p(x, x)$, *is a quasi-metric on* X *such that* $\tau(p) = \tau(d_p)$.

Lemma 1.3. [5] *A dualistic partial metric space* (X, p) *is complete if and only if the metric space* $(X, (d_p)^s)$ *is complete . Furthermore* $lim_{n\to\infty}(d_p)^s(a, x_n) = 0$ *if and only if* $p(a, a) = lim_{n\to\infty}p(a, x_n) = lim_{n,m\to\infty}p(x_n, x_m)$.

Before stating our main results we establish some (essentially known) correspondences between dualistic partial metric spaces and quasi-metric spaces. Our basic references for quasi-metric spaces are [3] and [4] and for $\epsilon-$ fixed point is [6].
Each quasi-metric d on X generates a T_0-topology $T(d)$ on X which has as a base the family of open d-balls $\{B_d(x, \epsilon), x \in X, \ \epsilon > 0\}$, where $B_d(x, \epsilon) = \{y \in X : \ d(x, y) < \epsilon\}$ for all $x \in X$ and $\epsilon > 0$.
If d is a quasi-metric on X, then the function d^s defined on $X \times X$ by $d^s(x, y) = max\{d(x, y), d(y, x)\}$, is a metric on X.

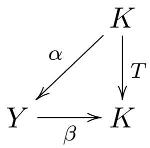
Theorem 1.4. [6] *Let* (X, p) *be a dualistic partial metric space and* $T : X \to X$ *be a map,* $x_0 \in X$ *and* $\epsilon > 0$ *. If* $d_p(T^n(x_0), T^{n+k}(x_0)) \to 0$ *as* $n \to \infty$ *for some* $k > 0$, *then* T^k *has an* $\epsilon-$ *fixed point.*

2. Main Results

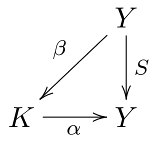
In this section, we give some results on fixed point and $\epsilon-$ fixed point in dualistic partial metric space and its diameter.

Definition 2.1. *An open ball for a dualistic partial metric* $p : X \times X \to \mathbb{R}$ *is a set of the form* $B^p_\epsilon(x) ::= \{y \in X : \ p(x, y) < \epsilon\}$ *for each* $\epsilon > 0$ *and* $x \in X$.

Theorem 2.2. *Let (X, p) be a dualistic partial metric space and K, Y be subsets of X. Also, let $\alpha : K \to Y$ and $\beta : Y \to K$ be two maps. Then $T = \beta\alpha : K \to K$ has a fixed point if and only if $S = \alpha\beta : Y \to Y$, has a fixed point. In other words, given the commutative diagrams:*



and



we have: $F(T) \neq \varnothing \Leftrightarrow F(S) \neq \varnothing$.

Proof. If y_0 is a fixed point of $\beta\alpha$ then it follows that $\alpha(y_0) = \alpha\beta[\alpha(y_0)]$. □

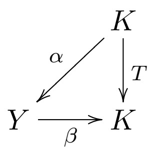
Definition 2.3. [6] *Let (X, p) be a dualistic partial metric space and $T : X \to X$ be a map. Then $x_0 \in X$ is $\epsilon-$ fixed point for T if*

$$d_p(Tx_0, x_0) \leqslant \epsilon.$$

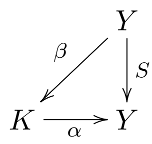
We say T has the ϵ-fixed point property if for some $\epsilon > 0$, $AF(T) \neq \emptyset$ where

$$AF(T) = \{x_0 \in X : \ d_p(Tx_0, x_0) \leqslant \epsilon\}.$$

Theorem 2.4. *Let (X, p) be a dualistic partial metric space and K, Y be subsets of X. Also, let $\alpha : K \to Y$ and $\beta : Y \to K$ be two maps and $AF(T) = AF(\alpha)$. Then $T = \beta\alpha : K \to K$ has an approximate fixed point if and only if $S = \alpha\beta : Y \to Y$, has an approximate fixed point. In order words, given the commutative diagrams:*



and



we have: $AF(T) \neq \varnothing \Leftrightarrow AF(S) \neq \varnothing$.

Proof. Since $AF(T) \neq \varnothing$, by Definition 2.3:

$$\begin{aligned} d(Ty_0, y_0) \leqslant \epsilon &\Leftrightarrow d(\beta\alpha(y_0), y_0) \leqslant \epsilon \\ &\Leftrightarrow d(\alpha[\beta\alpha(y_0)], \alpha(y_0)) \leqslant \epsilon \\ &\Leftrightarrow d(Sy_0, y_0) \leqslant \epsilon. \end{aligned}$$

Thus $AF(T) \neq \varnothing \Leftrightarrow AF(S) \neq \varnothing$. $\square$

Theorem 2.5. *Let (X, p) be a complete dualistic partial metric space and $T : X \to X$ be a map such that for all $x, y \in X$*

$$p(Tx, Ty) \leqslant cp(x, y) \ ; \ 0 \leqslant c < 1.$$

Then T has a unique fixed point u, and $T^n(x) \to u$ as $n \to \infty$ for each $x \in X$.

Proof. We shall show that for any given $x \in X$, the sequence $\{T^n(x)\}$ of iterates convergent to a fixed point. For this purpose, first of all observe that $p(Tx, T^2x) \leqslant cp(x, Tx)$ and by induction, $p(T^n x, T^{n+1}x) \leqslant c^n p(x, Tx)$ for all $n > 0$. Thus, for any $n > 0$ and any $k > 0$, we have

$$\begin{aligned} d_p(T^n(x), T^{n+k}(x)) &\leqslant \sum_{i=n}^{n+k-1} d_p(T^i(x), T^{i+1}(x)) \\ &\leqslant (c^n + \cdots + c^{n+k-1})(p(x, T(x)) - p(x, x)) \\ &\leqslant \frac{c^n}{1-c}(p(x, T(x)) - p(x, x)) \\ &= \frac{c^n}{1-c} d_p(x, T(x)). \end{aligned}$$

Since $c < 1$, then $c^n \to 0$. So by Lemma 1.3, $\{T^n(x)\}$ is a cauchy sequence in (X, d_p). Hence $T^n(x) \to u$ for some $u \in X$. By continuity of T, we should have $T(T^n(x)) \to Tu$. But $\{T^{n+1}(x)\}$ is a subsequence of $\{T^n(x)\}$, so $Tu = u$ and u is a fixed point for T. Therefore, we have shown that for each $x \in X$, the limit of the sequence $\{T^n(x)\}$ exists and is a fixed point, since we will show that T has at most one fixed point, and so every sequence $\{T^n(x)\}$ should be convergent to the same point. At the end we show the uniqueness of the fixed point of T : for if $T(x_0) = x_0$ and $T(y_0) = y_0$. Then $x_0 \neq y_0$ gives the contradiction:

$$\begin{aligned} d_p(x_0, y_0) &= d_p(T(x_0), T(y_0)) \\ &= p(T(x_0), T(y_0)) - p(T(x_0), T(x_0)) \\ &\leqslant c(p(x_0, y_0) - p(x_0, x_0)) \\ &< p(x_0, y_0) - p(x_0, x_0) \\ &= d_p(x_0, y_0). \quad \Box \end{aligned}$$

Corollary 2.6. *Let (X, p) be a complete dualistic partial metric space and $B_p = B_p(y_0, r) = \{y : d_p(y, y_0) < r\}$. Let $T : B_p \to X$ be a map such that for all $x, y \in X$*

$$p(Tx, Ty) \leqslant cp(x, y) \ ; \ 0 < c < 1. \tag{1}$$

If $d_p(Ty_0, y_0) < (1 - c)r$, then T has a fixed point.

Proof. Choose $\epsilon > r$, so that $d_p(Ty_0, y_0) \leqslant (1-c)r < (1-c)\epsilon$. We show that T maps the closed ball $K = \{y : d_p(y, y_0) \leqslant \epsilon\}$ into itself: for if $y \in K$, then

$$\begin{aligned} d_p(T(y), y_0) &\leqslant d_p(T(y), T(y_0)) + d_p(T(y_0), y_0) \\ &\leqslant cp(y, y_0) + (1 - c)\epsilon \\ &\leqslant c\epsilon + \epsilon - c\epsilon = \epsilon. \end{aligned}$$

Since K is complete and $T : K \to K$ satisfies in (1) thus by Theorem 2.5, T has a fixed point. $\Box$

Theorem 2.7. *Let (X, p) be a complete dualistic partial metric space and $T : X \to X$ be a map , not necessarily continuous. For each $\epsilon > 0$ there is a $\theta(\epsilon) > 0$ such that if $d_p(x, Tx) < \theta(\epsilon)$, then $T[B_d(x, \epsilon)] \subset B_d(x, \epsilon)$. Also, if $d_p(T^n(p_0), T^{n+1}(p_0)) \to 0$ for some $p_0 \in X$, the sequence $\{T^n(p_0)\}$ converges to a fixed point of T.*

Proof. We consider $T^n(p_0) = x_n$. First we show that $\{x_n\}$ is a Cauchy sequence. since $d_p(x_N, Tx_N) < \theta(\epsilon)$, we have $T[B_d(x_N, \epsilon)] \subset B_d(x_N, \epsilon)$. So $Tx_N = x_{N+1} \in B(x_N, \epsilon)$ and, by induction, $Tx_N = x_{N+k} \in B(x_N, \epsilon)$ for all $k \geqslant 0$. Thus, $d_p(x_m, x_n) < 2\epsilon$ for all $m, n \geqslant N$ and $\{x_n\}$ is Cauchy sequence. Therefore it converges to some $x_0 \in X$. Now we show that x_0 is a fixed point for T. Let $d_p(x_0, Tx_0) = b > 0$, we can choose $x_n \in B(x_0, \frac{b}{3})$ such that $d_p(x_n, x_{n+1}) < \theta\frac{b}{3}$: we have $T[B_d(x_N, \frac{b}{3})] \subset B_d(x_N, \frac{b}{3})$ by hypothesis, so $Tx_0 \in B(x_n, \frac{b}{3})$. But this is impossible because $d_p(Tx_0, x_n) \geqslant d_p(Tx_0, x_0) - d_p(x_n, x_0) \geqslant \frac{2b}{3}$. Thus $Tx_0 \notin B(x_n, \frac{b}{3})$ and so $d_p(x_0, Tx_0) = 0$. $\square$

Theorem 2.8. *Let (X, p) be a complete dualistic partial metric space and $T : X \to X$ be a map satisfying*

$$p(Tx, Ty) \leqslant \eta[p(x, y)],$$

where $\eta : \mathbb{R}^+ \to \mathbb{R}^+$ is any nondecrasing (not necessarily continuous) function such that $\eta^n(t) \to 0$ as $n \to \infty$ for each $t > 0$. Then T has a unique fixed point p_0, and $T^n(x) \to p_0$ as $n \to \infty$ for $x \in X$.

Proof. Observe that $\eta(t) < t$ for each $t > 0$, for if $t \leqslant \eta(t)$ for some $t > 0$, then monotonicity of η gives that $\eta(t) \leqslant \eta[\eta(t)]$ and by induction, $t \leqslant \eta^n(t)$ for all $n > 0$. So we have $p(Tx, T^2x) \leqslant cp(x, Tx)$ and by induction $p(T^n x, T^{n+1}x)| \leqslant c^n p(x, Tx)$ for all $n > 0$. Fix $x \in X$. Then clearly for each $x \in N$

$$|p(T^n x, T^n x)| \leqslant c^n |p(x, x)|$$

and

$$|p(T^n x, T^{n+1}x)| \leqslant c^n |p(x, Tx)|.$$

Also,

$$d_p(T^n x, T^{n+1}x) + p(T^n x, T^n x) = p(T^n x, T^{n+1}x).$$

Hence we deduce that

$$d_p(T^n x, T^{n+1}x) + p(T^n x, T^n x) \leqslant c^n |p(x, Tx)|.$$

Thus we get

$$\begin{aligned} d_p(T^n(x), T^{n+1}(x)) &\leqslant \eta^n |p(x,Tx)| - |p(T^n x, T^n x)| \\ &\leqslant \eta^n |p(x,Tx)| + |p(T^n x, T^n x)| \\ &\leqslant \eta^n (|p(x,Tx)| - |p(x,x)|) \\ &\leqslant \eta^n [d_p(x,Tx)]. \end{aligned}$$

So $d_p(T^n(x), T^{n+1}(x)) \to 0$ as $n \to \infty$ for each $x \in X$. Now let $\epsilon > 0$ be given, and choose $\theta(\epsilon) = \epsilon - \eta(\epsilon)$; if $d_p(x,Tx) < \theta(\epsilon)$, then for any $x_0 \in B_d(x,\epsilon)$ we have

$$d_p(Tx_0, x) \leqslant d_p(Tx_0, Tx) + d_p(Tx, x) < \eta[d_p(x_0, x)] + \theta(\epsilon) \leqslant \eta(\epsilon) + \epsilon - \eta(\epsilon) = \epsilon.$$

So $Tx_0 \in B_d(x,\epsilon)$. Hence by Theorem 2.7, T has a fixed point. The remainder of the proof is obvious. $\square$

Theorem 2.9. *Let (X,p) be a complete dualistic partial metric space and $T : X \to X$ be a map satisfying*

$$p(Tx,Ty) \leqslant \beta(x,y) p(x,y),$$

where $\beta : X \times X \to \boldsymbol{R}^+$ has the property for any closed interval $[a,b] \subset \boldsymbol{R}^+ - \{\not\vdash\}$,

$$sup\{\beta(x,y) : \ a \leqslant d_p(x,y) \leqslant b\} = \lambda(a,b) < 1.$$

Then T has an unique fixed point p, and $T^n(x) \to p$ as $n \to \infty$ for each $x \in X$.

Proof. For $x \in X$, the sequence $\{d_p(T^n(x), T^{n+1}(x))\}$ is nonincreasing, therefore it is convergent to some $a \geqslant 0$. We should have $a = 0$: otherwise, $d_p(T^n(x), T^{n+1}(x)) \in [a, a+1]$ for all large n; then by choosing n and $q = \lambda(a, a+1)$, by induction we have

$$a \leqslant d_p(T^{n+k}(x), T^{n+k+1}(x)) \leqslant q^k d_p(T^n(x), T^{n+1}(x)) \leqslant q^k (a+1)$$

for all $k > 0$, but $q < 1$, and is a contradiction. Now, suppose $\epsilon > 0$, $\lambda = \lambda(\frac{\epsilon}{2}, \epsilon)$ and choose $\theta = min\{\frac{\epsilon}{2}, \epsilon(1-\lambda)\}$. Let $d_p(x,Tx) < \theta(\epsilon)$ and $x_0 \in B_p(x,\epsilon)$ then

$$d_p(Tx_0, x) \leqslant d_p(Tx_0, Tx) + d_p(Tx, x).$$

If $d(x_0, x) < \frac{\epsilon}{2}$: then

$$d_p(Tx_0, x) \leqslant d_p(x_0, x) + d_p(Tx, x) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon;$$

and if $\frac{\epsilon}{2} \leqslant d(x_0, x) < \epsilon$: then

$$d_p(Tx_0, x) \leqslant \beta(x, y)d_p(x_0, x) + d_p(Tx, x) < \lambda\epsilon + (1 - \lambda)\epsilon = \epsilon.$$

Hence, $T[B_p(x, \epsilon) \subset B_p(x, \epsilon)$, and by Theorem 2.7, T has a fixed point. The remainder of the proof is obvious. $\square$

Proposition 2.10. *Let (X,p) be a dualistic partial metric space and $T : X \to X$ be a map such that T is asymptotic regular, i.e., for all $x \in X$*

$$d_p(T^n(x_0), T^{n+1}(x_0)) \to 0 \text{ as } n \to \infty.$$

Then T has an $\epsilon-$ fixed point.

Proof. Since $d_p(T^n(x_0), T^{n+1}(x_0)) \to 0$ as $n \to \infty$ then for $\epsilon > 0$, there exists $n_0 > 0$ such that for all $n \geqslant n_0$,

$$d_p(T^n(x_0), T^{n+1}(x_0)) < \epsilon.$$

Then $d_p(T^{n_0}(x_0), T(T^{n_0}(x_0))) < \epsilon$. Therefore $T^{n_0}(x_0)$ is an $\epsilon-$ fixed point of T. $\square$

Theorem 2.11. *Let T be a mapping of a dualistic partial metric space (X, p) into itself such that*

$$|p(Tx, Ty)| \leqslant c|p(x, y)| \quad 0 < c < d(\alpha(y_0), y_0)$$

for all $x, y \in X$, and $\epsilon > 0$. Then T^k has a ϵ-fixed point, for all k.

Proof. Fix $x \in X$. It is clear that for each $x \in N$

$$|p(T^n x, T^n x)| \leqslant c^n |p(x, x)|$$

also

$$|p(T^n x, T^{n+1} x)| \leqslant c^n |p(x, Tx)|,$$

and

$$d_p(T^n x, T^{n+1} x) + p(T^n x, T^n x) = p(T^n x, T^{n+1} x).$$

We deduce that

$$d_p(T^n x, T^{n+1} x) + p(T^n x, T^n x) \leqslant c^n |p(x, Tx)|.$$

Hence

$$\begin{aligned} d_p(T^n(x), T^{n+1}(x)) &\leqslant c^n |p(x, Tx)| - |p(T^n x, T^n x)| \\ &\leqslant c^n |p(x, Tx)| + |p(T^n x, T^n x)| \\ &\leqslant c^n (|p(x, Tx)| + |p(x, x)|). \end{aligned}$$

Therefore for $k, n \in N$

$$\begin{aligned} d_p(T^n(x), T^{n+k}(x)) &\leqslant d_p(T^n(x), T^{n+1}(x)) + ... + d_p(T^{n+k-1}(x), T^{n+k}(x)) \\ &\leqslant (c^n + ... + c^{n+k-1})(|p(x, Tx)| + |p(x, x)|) \\ &\leqslant \frac{c^n}{1-c}(|p(x, Tx)| + |p(x, x)|) \\ &\leqslant \frac{c^n}{1-c}[d_p(x, Tx)]. \end{aligned}$$

Thus $\lim_{n\to\infty} d_p(T^{n+k}(x), T^n(x)) = 0$ *as* $n \to \infty$. Therefore by Proposition 2.10 T^k has an ϵ-fixed point. $\square$

If we take $T : X \to X$ in Theorem 2.2 of [7], we have the following corollary.

Corollary 2.12. *Let (X,p) be a dualistic partial metric space and* $T : X \to X$ *be a mapping and* $\epsilon > 0$. *Also, let*

$$d_p(Tx, Ty) \leqslant \alpha d_p(x, y) + \beta[d_p(x, Tx) + d_p(y, Ty)]$$

for all $x, y \in X$, *where* $\alpha, \beta \geqslant 0$ *and* $\alpha + 2\beta < 1$. *Then* T *has a* ϵ*-fixed point.*

Example 2.13. Let $X = (-\infty, 2]$, and let p be the dualistic metric on X given by

$$p(x, y) = Max\{x, y\}$$

for all $x, y \in X$.
Let T be the mapping from X into itself defined by $T(x) = x - 1$, for all $X = (-\infty, 2]$. It is immediate to see that

$$p(T(x), T(y)) \leqslant \frac{1}{2} p(x, y)$$

for all $x, y \in X$. However T does not have any fixed point. But by Proposition 2.10, for some $\epsilon > 0$, T has a ϵ-fixed point.

Definition 2.14. *Let (X, p) be a dualistic partial metric space, $T : X \to X$, be continues map and $\epsilon > 0$. We define diameter $AF(T)$ by*

$$diam AF(T) = \sup\{d_p(x, y) : \quad x, y \in AF(T)\}.$$

If we take $T : X \to X$ in Theorem 2.8 of [7], we have the following corollary.

Corollary 2.15. *Let $T : X \to X$ and $\epsilon > 0$. If there exists $\alpha \in [0, 1]$ such that for all $x, y \in X$*

$$d_p(Tx, Ty) \leqslant \alpha d_p(x, y),$$

then

$$diam AF(T) \leqslant \frac{2\epsilon}{1 - \alpha}.$$

References

[1] I. Altun and A. Erduran, Fixed point theorems for monotone mappings on partial metric spaces, *Fixed Point Theory and Applications*, (2011).

[2] M. Berinde, Approximate fixed point Theorems, *Stud. Univ. Babes Bolyai, Math.,* 51 (1) (2006), 11-25.

[3] P. Fletcher and W. F. Lindgren, *Quasi-Uniform Spaces,* Marcel Dekker, New York, (1982).

[4] H. P. A. Kunzi, Handbook of the History of General Topology, ch. Non-symmetric distances and their associated topologies: About the origins of basic ideas in the area of asymmetric topology, *Kluwer Acad.* Publ., (2001), 853-968.

[5] S. G. Matthews, Partial metric topology, In Proceedings of the 11th Summer Conference on General Topology and Applications, The New York Academy of Sciences, *Gortham, Me, USA*, Augusts, 728 (1995), 183-197.

[6] S. A. M. Mohsenalhosseini, H. Mazaheri, M. A. Dehghan, and A. Zareh, Fixed point for Partial Metric Spaces, *ISRN Applied Mathematics*, Special section, (2011), 1-6.

[7] S. A. M. Mohsenalhosseini, H. Mazaheri, and M. A. Dehghan, Approximate best proximity pairs in metric space, *Abstract and Applied Analysis*, (2011).

[8] S. Oltra and O. Valero, Banachs fixed point Theorem for Partial Metric Spaces, *Rend. Istit. Mat. Univ. Trieste*, 5 (2004), 17-26.

[9] S. J. O'Neill, Partial metrics, valuations and domain theory, Proc. 11th Summer Conference on General Topology and Applications, *Ann. New York Acad. Sci.,* 806 (1996), 304-315.

Seyed Ali Mohammad Mohsenalhosseini
Department of Mathematics
Faculty of Mathematics
Assistant Professor of Mathematics
Vali-e-Asr University
Rafsanjan, Iran
E-mail: amah@vru.ac.ir

Hamid Mazaheri
Department of Mathematics
Faculty of Mathematics
Associate Professor of Mathematics
Yazd University
Yazd, Iran
E-mail: hmazaheri@yazduni.ac.ir