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Original Research Paper

Study on a New Family of Close-to-Convex Functions in a Vertical Strip Involving Subordination and Convolution

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Abstract. In this present paper, we consider a subclass of close-to-convex functions associated with a vertical strip domain. We obtain several results concerned with integral coefficient inequalities, convolutions and representations for functions belonging to this class $\mathbf{MC_c}$. Furthermore, we consider the Fekete-Szego, inclusion relations and radius problems involving certain classes of closto-convexity related to vertical strip domains. Some well-known results appears as special cases from our derived results.

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1 Introduction

Let $\mathbb{A}$ represent the class of the function $\xi(z)$ having the series form

$$\xi(z) = z + \sum_{\ell=2}^{\infty} \mathfrak{a}_\ell z^\ell \tag{1}$$

which is holomorphic(analytic) and univalent(one-one) in the open unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$. Let ξ be given in (1) and $g \in \mathbb{A}$ given as $g(z) = z + \sum_{\ell=2}^{\infty} b_\ell z^\ell$ the convolution of which is illustrated by

$$(\xi * g)(z) = z + \sum_{\ell=2}^{\infty} a_\ell b_\ell z^\ell.$$

A function $\xi \in \mathbb{A}$ fulfills the condition $R\left(\frac{z\xi'(z)}{\xi(z)}\right) > \varphi \quad (z \in D)$ is said to be starlike of order φ $(0 \leq \varphi < 1)$. $S^*(\varphi)$ represents the class of all such functions. It is commonly understood that $S^*(\varphi) \subset S^*(0) = S^* \subset S$.

Consider that $0 \leq \varphi, \vartheta < 1$. If there exists a function g $\in S^*(\varphi)$ such that the inequality $\Re\left(\frac{z\xi'(z)}{g(z)}\right) > \varphi \quad (z \in D)$ is satisfied, then the function $\xi \in \mathbb{A}$ is called *close-to-convex* of order φ and type ϑ. We used $C_c(\varphi ,\vartheta)$ to represent the class that consists of any function that is *close-to-convex* of order φ and type ϑ. This class is investigated by Libera [12]. More specifically, when ϑ = 0 we have $C_c(\varphi,0)$ = $C_c(\varphi)$ of *close-to-convex* function of order φ, also we get $C_c(0,0)$ = C_c class of *close-to-convex* function investigated by Kaplan [7]. it is well-known that $S^* \subset C_c \subset$ S. The ξ and h are two holomorphic functions in D. We say that function ξ in D is subordinate to function h and write as

$$\xi(z) \prec h(z) \quad (z \in D)$$

if there exists a Schwarz funtion $v(z)$ such that $\xi(z) = h\big(v(z)\big) \quad (z \in D)$. It is known that if h is univalent(one-one) in$D$ then

$$\xi(z) \prec h(z) \Longleftrightarrow \xi(0) \;\; = \;\; h(0) \;\; and \;\; \xi(D) \subset h(D).$$

A function $\xi \in \mathbb{A}$ is said to be strongly close-to-convex of order $0 \leq \gamma < 1$ if there is a function $g(z) \in S^*$ such that ([4])

$$\left| arg\left(\frac{z\xi'(z)}{g(z)}\right)\right| \leq \frac{\pi}{2}\gamma \quad (z \in D).$$

We denote it by $SC_c(\gamma)$. While a function $\xi \in \mathbb{A}$ is said to be parabolic close-to-convex function ([9]) if there is a function g ∈ S* such that

$$\left|\frac{z\xi'(z)}{\xi(z)} - 1\right| \leq R\left(\frac{z\xi'(z)}{g(z)}\right) \quad (z \in D).$$

We denote it by PC_c.
Srivastava et al. [19] investigated the class of close-to-convex functions associated with the lemniscate of Bernoulli denoted by CL_c. A function $\xi \in \mathbb{A}$ is said to be the class CL_c if there exists a function $g \in S*$ such that

$$\left(\frac{z\xi'(z)}{g(z)}\right) \prec \sqrt{1+z} \quad (z \in D).$$

Recently Karger et al. [8] developed the holomorphic function U_σ and vertical strip Φ_σ defined as follows:

$$U_\sigma(z) = \frac{1}{2\iota\sin(\sigma)} \log\left(\frac{1+e^{\iota\sigma}z}{1+e^{-\iota\sigma}z}\right) \ (z \in D) \tag{2}$$

and

$$\Phi_\sigma = \left\{v \in C : \frac{\sigma-\pi}{2\sin(\sigma)} < \Re(v) < \frac{\sigma}{2\sin(\sigma)}\right\}, \tag{3}$$

where $(\frac{\pi}{2} \leq \sigma\pi)$.They also introduced the following class $M(\sigma)$:
$M(\sigma) = \left\{\xi \in \mathbb{A} : [1 + \frac{\sigma-\pi}{2\sin(\sigma)} < \Re\left(\frac{z\xi'(z)}{\xi(z)}\right) < 1 + \frac{\sigma}{2\sin(\sigma)}]\right\}$.
It is clear that the function U_σ given in (2) is convex and univalent (one-to-one) in D. Furthermore, U_σ maps D onto Φ_σ when $(\frac{\pi}{2} \leq \sigma < \pi)$. In other choices of σ, the image could be a triangle, a half strip or a trapezium (see [5]).

Note that the following form may be used to write the function U_σ:

$$U_\sigma(z) = z + \sum_{\ell=2}^{\infty} B_\ell(\sigma)z^\ell \qquad (\frac{\pi}{2} \leq \sigma < \pi; z \in D),$$

where,

$$B_{\ell}(\sigma) = (-1)^{(\ell-1)} \frac{\sin(\ell\sigma)}{\ell \sin(\sigma)} \qquad (\frac{\pi}{2} \leq \sigma < \pi; \ell \in N). \tag{4}$$

In the last few years, significant results have been reported on the class of normalized holomorphic functions$\xi \in \mathbb{A}$that map D onto a vertical strip. For related study one can study [2],[6], [11], [13], [15], [16], [18], [20], [22].

A Motivation: Motivated by above works, we will define the following family of close-to-convex functions associated with the vertical strip Φ_σ due to its interesting geometry.

Definition 1.1. *A function $\xi \in \mathbb{A}$ is said to belong to the class $\mathbf{MC}(\sigma)$, where $(\pi/2 \leq \sigma < \pi)$ if it there exists a function $g \in M(\sigma)$ such that:*

$$1 + \frac{\sigma - \pi}{2\sin(\sigma)} < \Re\left(\frac{z\xi'(z)}{g(z)}\right) < 1 + \frac{\sigma}{2\sin(\sigma)} \quad (z \in D)$$

.

Remark 1.1. *From the article [8], we conclude that $1 - \frac{\pi}{4} \leq 1 + \frac{\sigma-\pi}{2\sin(\sigma)} < \frac{1}{2}$ and $1+\frac{\sigma}{2\sin(\sigma)} \geq 1+\frac{\pi}{4}$ $(\pi/2 \leq \sigma < \pi)$. Clearly, $C_c \supset \mathbf{MC_c}(\sigma)$ $(\frac{\pi}{2} \leq \sigma < \pi)$ and $C_c(\frac{4-\pi}{4}, \frac{4+\pi}{4}) \supset \mathbf{MC_c}(\pi/2)$, where the class $C_c(\varphi, \eta), 0 \leq \varphi < 1 < \eta$, was recently introduced by S. Bulut [3].*

We aim to prove certain coefficient bounds, including Fekete-Szegö type results, inclusion relations and radius problems involving functions in the class $MC_c(\sigma)$.

2 Main Results

First, we need the following lemmas to prove the main results.

Lemma 2.1. *Let $\xi \in \mathbf{MC_c}(\sigma)$ $\quad(\frac{\pi}{2} \leq \sigma < \pi)$. Then*

$$\left(\frac{z\xi'(z)}{g(z)} - 1\right) \prec U_\sigma = \frac{1}{2\iota\sin(\sigma)} \log\left(\frac{1 + e^{(\iota\sigma)}z}{1 + e^{(-\iota\sigma)}z}\right) \quad (z \in D). \tag{5}$$

Proof. From (3), we see that

$$v(z) = \left\{\frac{z\xi'(z)}{g(z)}\right\} - 1$$

lies in a strip Φ_a and it is known that $U_\sigma(D) = \Phi_a$. Because $U_\sigma(z)$ is univalent then by the subordination principle we get (5). $\square$

Lemma 2.2. *[14]*
Let h be convex in D and let G : $D \longrightarrow \mathbb{C}$, with $R(G(z)) > 0$. If G is holomorphic in D, then

$$\begin{aligned} q(z) + G(z)zq'(z) &\prec h(z) \\ \Rightarrow \qquad q(z) &\prec h(z). \end{aligned} \tag{6}$$

Lemma 2.3. *[17]*
Suppose that u(z) is holomorphic, univalen and maps D onto a convex domain. Assume that u(z) given by

$$u(z) = \sum_{\ell=1}^{\infty} C_\ell z^\ell$$

is holomorphic and univalent in D. Also assume that

$$s(z) = \sum_{\ell=1}^{\infty} A_\ell z^\ell$$

is holomorphic in D and that the subordination relation

$$s(z) \prec u(z) \qquad (z \in D)$$

is satisfied. Then,

$$|A_\ell| \leq |C_1| \qquad (z \in N).$$

Lemma 2.4. *[1]*
If $p(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + c_4 z^4 + \ldots$ is a function with a positive real part in D and $\Psi \in \mathbb{R}$(real number) then

$$|c_2 - \Psi c_2^2| \leq 2max\{1; |2\Psi - 1|\}.$$

Theorem 2.5. *A function $\xi \in \mathbf{MC_c}(\sigma)$ $(\frac{\pi}{2} \leq \sigma < \pi)$ if and only if*

$$\xi(z) = \int_0^z \left[\frac{g(x)}{x} + \frac{g(x)}{2\iota\, x\, \sin(\sigma)} \log\left(\frac{1+e^{\iota\sigma}x}{1+e^{-\iota\sigma}x}\right)\right] dx \quad (z \in D). \tag{7}$$

Proof. For $\xi \in \mathbf{MC_c}(\sigma)$, we know from Lemma 2.1 that relation (5) holds. It follows that

$$\frac{z\xi'(z)}{g(z)} - 1 = \frac{1}{2\iota \sin(\sigma)} \log\left(\frac{1+e^{\iota\sigma}v(z)}{1+e^{-\iota\sigma}v(z)}\right) \quad (z \in D), \tag{8}$$

where the Schwarz function $v(z)$ is analytic in D with $v(0) = 0$ and $\left|v(z)\right| < 1$ ($z \in$ D). We next see from (8) that

$$\frac{z\xi'(z)}{g(z)} - \frac{1}{z} = \frac{1}{2\iota z \sin(\sigma)} \log\left(\frac{1+e^{\iota\sigma}v(z)}{1+e^{-\iota\sigma}v(z)}\right) \quad (z \in D)$$

$$\xi'(z) - \frac{g(z)}{z} = \frac{g(z)}{2\iota\ z\ \sin(\sigma)} \log\left(\frac{1+e^{\iota\sigma}z}{1+e^{-\iota\sigma}z}\right) \quad (z \in D) \tag{9}$$

$$\xi'(z) = \frac{g(z)}{z} + \frac{g(z)}{2\iota z \sin(\sigma)} \log\left(\frac{1+e^{\iota\sigma}z}{1+e^{-\iota\sigma}z}\right) \quad (z \in D) \tag{10}$$

Integrating both sides from 0 to z, we get the desired result. □

Theorem 2.6. *A function $\xi \in \mathbf{MC_c}$ $(\sigma)(\frac{\pi}{2} \leq \sigma < \pi)$, if and only if*

$$\left\{\xi(z)*\frac{z}{(1-z)^2}\right\}-\left\{1+\frac{1}{2\iota \sin(\sigma)} \log\left(\frac{1+e^{\iota(\phi-\sigma)}}{1+e^{\iota(\phi-\sigma)}}\right)\right\}*g(z) \neq 0 \quad (z \in D), \tag{11}$$

where "$*$" denote the "Hadamard product", $(0 \leq \phi < 2\pi)$ and $\phi - \sigma = \pi$.

Proof. Let $\xi \in \mathbf{MC_c}$ (σ). By using Lemma 2.1, we observe that (5) holds.

$$\Longrightarrow \quad \frac{z\xi'(z)}{g(z)} \neq 1 + \frac{1}{2\iota \sin(\sigma)} \log\left(\frac{1+e^{(\iota\sigma)}e^{(\phi)})z}{1+e^{(-\iota\sigma)}e^{(\phi)}}\right), \tag{12}$$

where $(0 \leq \phi < 2\pi)$ and $(\phi - \sigma \neq \pi)(z \in D)$. The condition (12) can now be written as follows:

$$\Longrightarrow \quad \frac{z\xi'(z)}{g(z)} - \left[1 - \frac{1}{2\iota \sin(\sigma)} \log\left(\frac{1+e^{(\iota\sigma)}e^{(\phi)})}{1+e^{(-\iota\sigma)}e^{(\phi)}}\right)\right] g(z) \neq 0 \tag{13}$$

where $(0 \leq \phi < 2\pi)$ and $(\phi - \sigma \neq \pi)(z \in D)$. Note that,

$$g(z) = g(z) * \left(\frac{z}{1-z}\right) \quad and \quad z\xi'(z) = \xi(z) * \left(\frac{z}{(1-z)^2}\right). \tag{14}$$

Thus by (13) and (14), we obtain assertion (11) of Theorem 2.6.

□

Theorem 2.7. *Let $\xi \in A$ satisfy the subordination*

$$\frac{(z\xi'(z))'}{g'(z)} \prec 1 + U_\sigma(z) \quad (z \in D). \tag{15}$$

for $g \in M(\sigma)$. Then

$$\frac{z\xi'(z)}{g(z)} \prec 1 + U_\sigma(z) \quad (z \in D). \tag{16}$$

That is, $\xi \in \mathbf{MC_c}\ (\sigma)$.

Proof. Suppose that the function $p(z)$ such that

$$p(z) = \frac{z\xi'(z)}{g(z)} \ \ (z \in D). \tag{17}$$

Then, $\frac{\left(z\xi'(z)\right)'}{g'(z)} = p(z) + zp'(z)\frac{g(z)}{zg'(z)}$. Let $G(z) = \frac{g(z)}{zg'(z)}$. Hence,

$$p(z) + zp'(z)G(z) = \frac{\left(z\xi'(z)\right)'}{g'(z)} \prec U_\sigma(z) \quad (z \in D). \tag{18}$$

Note that

$$p(0) = 0 = U_\sigma(0) \quad and \quad \Re(1 + U_\sigma(z)) > 0 \quad (\frac{\pi}{2} \le \sigma < \pi; z \in U). \tag{19}$$

Moreover by using Lemma 2.2, we get

$$p(z) \prec U_\sigma(z), \quad \Longrightarrow \quad \frac{z\xi'(z)}{g(z)} \quad \prec \quad 1 + U_\sigma(z).$$

□

Theorem 2.8. *Consider the function*

$$\xi(z) = \sum_{\ell=2}^{\infty} a_\ell z^\ell \in \mathbf{MC_c}(\sigma).$$

Then, $|a_\ell| \le 1 \qquad (\ell \in N)$.

Proof. For given σ $(\frac{\pi}{2} \leq \sigma < \pi)$, we define the functions $q(z)$ and $p(z)$ by

$$q(z) = \frac{z\xi'(z)}{g(z)} \qquad (z \in D), \tag{20}$$

and

$$p(z) = 1 + \frac{1}{2\iota \sin(\sigma)} \log\left(\frac{1+e^{(\iota\sigma)}z}{1+e^{(-\iota\sigma)}z}\right) \qquad (z \in D). \tag{21}$$

Then the subordination (5) can be written as follows;

$$q(z) \prec p(z) \qquad (z \in D). \tag{22}$$

Note that the function p(z) defined by (21) is convex in D and has the form

$$p(z) = 1 + \sum_{\ell=1}^{\infty} B_\ell(\sigma) z^\ell \qquad (z \in D), \tag{23}$$

where $B_\ell(\sigma)$ is given by (4) and $q(z)$

$$q(z) = 1 + \sum_{\ell=1}^{\infty} A_\ell z^\ell \qquad (z \in D), \tag{24}$$

then by Lemma2.3 we see that the subordination relation (13) implies that

$$\left|A_\ell\right| \leq \left|B_1\right| \quad (\ell \in N). \tag{25}$$

Now (20) implies that

$$z\xi'(z) = g(z)q(z) \qquad (z \in D). \tag{26}$$

Then by equating the coefficient of z^ℓ both sides we get

$$a_\ell = \frac{1}{(\ell-1)}\left[A_{\ell-1} + b_2A_{\ell-2} + b_3A_{\ell-3} + ... + b_{\ell-1}A_1\right] \quad (\ell \in N\diagdown\{1\}). \tag{27}$$

A simple calculation combined with inequality (25) yields $|a_2 = |A_1| \leq 1$ and

$$\left|a_\ell\right| = \frac{1}{(\ell-1)}\left|A_{\ell-1} + b_2A_{\ell-2} + b_3A_{\ell-3} + ... + b_{\ell-1}A_1\right| \tag{28}$$

$$|a_\ell| \leq \frac{1}{(\ell-1)}\Big[|A_{\ell-1}| + |b_2||A_{\ell-2}| + |b_3||A_{\ell-3}| + ... + |b_{\ell-1}||A_1|\Big]. \quad (29)$$

Using 25 and $|b_k| \leq 1$ (see [21]), we have

$$|a_\ell| \leq |B_1| = 1. \quad (30)$$

Hence proved.

□

3 The Fekete-Szego problem

In this section , first give results on the Fekete-Szego problem involving the function class $\mathbf{MC_c}\,(\sigma)$. The basic mathod of the proof in the following theorem is similar to that used.

Theorem 3.1. *Let*

$$(\xi(z)) = 1 + \mathfrak{a}_2 z^2 + \mathfrak{a}_3 z^3 + \mathfrak{a}_4 z^4 + ...$$

be in $\mathbf{MC_c}\,(\sigma)$*. Then for* $\Lambda \in \mathbb{R}$*,*

$$|\mathfrak{a}_3 - \Lambda \mathfrak{a}_2^2| = \frac{1}{3|1-\varpi|} Max\{1; |2\Lambda_1 - 1|\}, \quad (31)$$

where

$\varpi = \frac{\sigma - \pi}{2\sin(\sigma)}$,

$\Lambda_1 = \frac{-3(1-\varpi)\Upsilon}{d_1^2} - \Lambda\frac{3}{4(1-\varpi)}$ *and*

$\Upsilon = \left[\frac{b_3}{3} + \frac{b_2 d_1}{3(1-\varpi)} - \Lambda\frac{b_2^2}{4} - \Lambda\frac{b_2 d_1}{2(1-\varpi)}\right].$

Proof. For $\upsilon \in \Phi_\sigma$, we have $\Re(\upsilon(z)) > \varpi$. That is

$$\left(\frac{\upsilon(z) - \varpi}{1-\varpi}\right) \in P,$$

where P denotes the class of functions with positive real part. Setting

$$\left(\frac{\upsilon(z) - \varpi}{1-\varpi}\right) = p(z) = 1 + \sum_{\ell=1}^{\infty} c_\ell z^\ell.$$

Then, $v(z)=1+\sum_{\ell=1}^{\infty}(1-\varpi)c_\ell z^\ell$. Let

$$v(z)=\frac{z\xi'(z)}{g(z)}-1 \qquad (z\in D) \tag{32}$$

where $g(z)=z+\sum_{\ell=2}^{\infty}b_\ell z^\ell \quad \in M(\sigma)$. Then, $(v(z)+1)g(z)=z\xi'(z)$. Now, $(v(z)+1)g(z)=\left(2+\sum_{\ell=1}^{\infty}(1-\varpi)c_\ell z^\ell\right)\left(z+\sum_{\ell=2}^{\infty}b_\ell z^\ell\right)$. In series form, we get

$$(v(z)+1)g(z)=\left(2+(1-\varpi)c_1z^1+(1-\varpi)c_2z^2+(1-\varpi)c_3z^3+...+(1-\varpi)c_\ell z^\ell+...\right)\times$$

$$\left(z+b_2z^2+b_3z^3+b_4z^4+...+b_\ell z^\ell+...\right)$$

$$\begin{aligned}&=2z+2b_2z^2+2b_3z^3+2b_4z^4+...\\&+(1-\varpi)c_1z^2+(1-\varpi)b_2c_1z^3+(1-\varpi)b_3c_1z^4+(1-\varpi)b_4c_1z^5+...\\&+(1-\varpi)c_2z^3+(1-\varpi)b_2c_2z^4+(1-\varpi)b_3c_2z^3z^5+(1-\varpi)b_4c_2z^6+...\end{aligned} \tag{33}$$

Also

$$z(\xi'(z))=z+2\mathfrak{a}_2z^2+3\mathfrak{a}_3z^3+...+\ell\mathfrak{a}_\ell z^\ell+... \tag{34}$$

Comparing the coefficients (33) and (34) of $z^2 and\ z^3$

$$\mathfrak{a}_2=\frac{1}{2}\left(2b_2+(1-\varpi)c_1\right)$$

$$\mathfrak{a}_3=\frac{1}{3}\left(2b_3+(1-\varpi)b_2c_1+(1-\varpi)c_2\right). \tag{35}$$

Let $\Lambda\in\ \mathbb{R}$ and

$$\mathfrak{a}_3-\Lambda\mathfrak{a}_2^2=\left[\frac{1}{3}\left(2b_3+(1-\varpi)b_2c_1+(1-\varpi)c_2\right)-\Lambda\left\{\frac{1}{2}\left(2b_2+(1-\varpi)c_1\right)\right\}^2\right]. \tag{36}$$

Then

$$\mathfrak{a}_3-\Lambda\mathfrak{a}_2^2=\left[\frac{2b_3}{3}+\frac{(1-\varpi)b_2d_1}{3}+\frac{(1-\varpi)d_2}{3}-\Lambda b_2^2-\Lambda\frac{(1-\varpi)b_2c_1}{2}-\Lambda\frac{(1-\varpi)^2c_1^2}{4}\right].$$

Let $\Upsilon = \left[\frac{2b_3}{3} + \frac{(1-\varpi)b_2 d_1}{3} - -\Lambda b_2^2 - \Lambda\frac{(1-\varpi)b_2 c_1}{2}\right]$. Then

$$\mathfrak{a}_3 - \Lambda\mathfrak{a}_2^2 = \left[\Upsilon + \frac{(1-\varpi)d_2}{3} - \Lambda\frac{(1-\varpi)^2 c_1^2}{4}\right]$$

$$\mathfrak{a}_3 - \Lambda\mathfrak{a}_2^2 = \frac{(1-\varpi)}{3}\left[3\frac{\Upsilon}{1-\varpi} + c_2 - \Lambda\frac{3(1-\varpi)c_1^2}{4}\right].$$

Let$\Lambda_1 = \frac{-3\Upsilon}{(1-\varpi)c_1^2} - \Lambda\frac{3(1-\varpi)}{4}$ and

$$\mathfrak{a}_3 - \Lambda\mathfrak{a}_2^2 = \frac{1}{3(1-\varpi)}\left[c_2 - \Lambda_1 c_1^2\right]. \tag{37}$$

Give modulus on both side (37) and applying the Lemma 2.4, we get

$$|\mathfrak{a}_3 - \Lambda\mathfrak{a}_2^2| \leq \frac{|1-\varpi|}{3} Max\{1; |2\Lambda_1 - 1|\}. \tag{38}$$

□

4 Conclusion

The problem of studying different classes of analytic functions is well-known specially with some nice geometrical description e.g starlikeness and convexity. The recent study of starlikeness and convexity with the geometry of vertical strip domain is given, see [8, 10, 11, 13]. Motivated by these works, in the present article we aim to study another geometric property known as close-to-convexity associated with domain of vertical strip. It is important to mention that some well known results appeared as special cases from our derived results. In concluding for future works one may investigate other geometrical description of analytic functions in a similar fashion. The results involving convolution techniques are also point of further interest and investigation in this direction.

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