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Original Research Paper

A Bi-Objective Model for The Collaborative Uncapacitated Arc Routing Problem Including Coalition Satisfaction

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Abstract. This paper deals with a carrier collaboration problem in which carriers form a coalition to share and exchange their transportation requests located on the arcs of the network. This problem has recently gained a significant attention to enhance the operational efficiency. However, the existing problem formulations are single objective optimization problems solely focusing on maximizing the profit and overlooking satisfaction criterion which is crucial to the coalition's long-term survival. This paper proposes a bi-objective integer linear programming model aiming at maximizing profit as well as coalition satisfaction. In order to ensure satisfaction among all members of the coalition, an efficient metric is proposed using proper mathematical norms

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(L_1-Norm and L_∞-Norm) , encouraging a well-distribution of satisfaction among members. Moreover, this paper shows how the carriers' overtime can be introduced in the problem formulation which could potentially improve the total profitability of the system and coalition satisfaction. The proposed models are tested on a publicly available data set of 1416 instances and the performance of the models is analyzed.

AMS Subject Classification: 90C27; 90C29; 90C90; 90B06; 91A12; 91A80

Keywords and Phrases: Multiple objective programming, Arc routing problem, Collaboration, Route balancing, Coalition satisfaction, Fair distribution.

1 Introduction

The transportation industry is highly competitive and companies require to maximize their efficiency level to stay in their respective businesses. Fierce competition reduces prices and hence profit margins have decreased to an extremely low level. As way to increase efficiency, these companies can set up collaborations [?]. The collaboration strategies have been recently enhanced by the developments in information and communication technology which has enabled carriers to share their transportation requests to increase the performance and profit of the network while reducing operational costs [?]. Besides increasing efficiency, collaborations also result in serving ecological goals. Transportation is considered as one of the major contributors of CO2 emissions. Thus, public authorities are motivating companies to collaborate not only to reduce emissions of harmful substances, but also to decrease road congestion and noise pollution. Furthermore, collaborations in logistics have been shown to ensure higher service levels, enhance capacities, gain market shares, and reduce the negative impacts of the bullwhip effect [?]. Recently, the collaboration topic has gained attention and many scholars have shown how this technique can improve: (i) the logistics performance of the carriers [?], (ii) the goods distribution network [?], and (iii) the economic, environmental, and social sustainability ([?] and [?]. Strategies used in the collaborative transportation system might consist of exchanging customers among carriers [?], sharing vehicles' capacity of various carriers with a similar depot [?], consolidating cargo of different carriers in single delivery routes [?], or providing loads in backhaul routes to decrease empty trips ([?] and [?]).

The collaboration uncapacitated arc routing problem (CUARP) was first introduced in [?]. The collaboration type is centralized in which the collaborative decisions are made by a central authority that has full information. Each carrier owns a depot and a set of customers. Each customer is expressed via an arc (for example newspaper delivery and postal service, meter reading, urban garbage collection, snow plowing, leisure and tourism, street cleaning [?]) and its service creates revenue. In their collaboration scheme, each carrier must serve a part of its customers and share the remaining part to be possibly served by other carriers. It is assumed that each carrier has one vehicle and the vehicle capacity is not relevant because of the small sizes of the consigned packages and documents, meaning the vehicles are uncapacitated [?]. In one variant, it is guaranteed that the profit collected by each carrier in the collaborating system is not lower than that in the case of working independently.

Many applications arise in private companies for which collaboration and competition are integrated to enhance their performance, such as mail and small packaging distribution and taxi services [?]. In profit-based applications, the main idea of collaboration is to use the capacity of all carriers to gain more profit compared to the case that they work independently. However, ignoring satisfaction criterion and merely focusing on profitability in a customer-based alliance seems to be unwise. In addition, satisfaction should be fairly distributed among coalition members, or otherwise, the coalition may tend to serve the demands of some specific carriers and potentially, endanger the coalition's long-term survival. Moreover, in real-world applications, the normal working time for carriers, including traveling time as well as service time, is limited to pre-defined quantities that can be embedded into the model through hard constraints. However, relaxing time constraints by extending the working time of carriers might bring benefits, such as profitability, to the coalition.

This paper proposes a new bi-objective model for collaborative uncapacitated arc routing problem which includes coalition satisfaction (B-CUARPS), in which carrier overtime is also allowed. In addition, an efficient and effective metric is proposed to implement this objective. In this metric, mathematical norms are integrated in such a way that a fair balance for satisfaction among coalition members is achieved. Another contribution of this paper is to provide the possibility of working overtime for carriers. Indeed, the model incorporates overtime into coalitions' profit that enables the carriers to make more profit within extended working time, if it is cost-benefit. The rest of the paper is organized as follows. In Section 2, we briefly review the relevant literature. The proposed model is formally described and formulated in Section 3. In order to solve the proposed bi-objective model, the augmented ε-constraint algorithm is described in Section 4. Some examples for clarifying the importance of our model and several computational experiments are reported in Section 5. Finally, section 6 concludes this work.

2 Related Works

The collaboration setting in routing problems is an effective strategy to reduce the transportation costs, enhance the capacity utilization, and improve the logistics efficiency. However, in the optimization field, only a few papers have proposed models aimed at optimizing the routes of the companies involved in a collaboration initiative [**?**]. In [**?**], the collaboration uncapacitated arc routing problem (CUARP) was introduced where carriers collaborate to improve their profits. The authors proposed two models. In the first model (of the two proposed ones), the goal is the maximization of the total profit of the coalition of carriers, regardless of the individual profit of each carrier achieved in the case they work independently. Then, a lower bound on the individual profit of each carrier is included that may represent the profit of each carrier in the non-collaboration setting. A collaborative version of a routing problem with profits was proposed in [**?**], where customers belonging to different partners in a logistic coalition are served in a single logistic operation with multiple vehicles. In [**?**], the authors introduced the shared customer collaboration vehicle routing problem (SCC-VRP). This problem aims at minimizing the total operational cost in a collaboration framework among the carriers for the service of the shared customers.

The collaborative multi-depot vehicle routing problem with pickups and deliveries (MDVRPPD) was provided in [?]. Each company is guaranteed to serve at least a minimum amount of its own customers, at least a fixed portion of other companies' customers while profit is bounded to be greater or equal than the profit obtained in a non collaborative setting. The potential advantages of collaboration among less than truckload carriers (LTL), i.e. the size of customer orders is less than the vehicle's capacity, in performing pickup and delivery tasks were studied in [?] with the objective of minimizing the total transportation cost. In another collaborative work ([?] (, each carrier has customers who are geographically spread over a specific region. Through establishing coalitions and collaborating, carriers share their distribution center in order to generate an optimal network and obtain more profit. The collaborative vehicle routing problem with time and service consistency and workload balance is studied in [?]. In this problem, carriers can exchange customers who must be regularly serviced. Carriers might wish to enter a collaboration only if a minimum market share can be guaranteed.

Some papers investigated more than one objective in the collaboration scheme. In [?], a collaborative vehicle routing problem involving shared carriers and depots (CVRP-SCD) was investigated. Four objectives, including quality, reliability, cost, and time is integrated to form a composite objective function. In [?], each individual member of the collaboration is able to set its desired set of objectives. Then, the question of whether only these individual objectives or a set of coalition objectives should be considered in the relevant collaborative optimization problem is addressed. The benefits and pitfalls of both methods are analyzed through applying them to a collaborative variant of the traveling salesman problem. A transportation resource sharing strategy is introduced in [?] to address the multi-depot green vehicle routing problem. A bi-objective model is proposed to minimize total carbon emission and operating cost while imposing piecewise penalty costs. Different categories used in the formulation of routing and collaboration and multi-objective problems can also be found in [?], [?], [?], [?], [?], and [?].

Recently, people have gradually become aware of environmental issues such as urban air pollution and global warming, which has led to more environmental protection efforts [?], [?]. Greenhouse gas (GHG) emissions are an important factor leading to these environmental problems [?], [?], [?]. Therefore, reducing GHG emissions has become an important international issue. GHG emissions mainly come from the exhaust emissions of transportation carriers, such as cars, trucks, airplanes, etc. [?]. The aims are the minimization of travel time and fuel consumption [?]. In addition, sustainability related indices such as social concerns in transportation and logistics (e.g. satisfaction, level of service, etc.) should also be considered [?].

Regarding the importance of time in our study on ARPs, a brief review of the related literature is presented. For many applications of ARPs, it is important from both the theoretical and practical perspectives that the services are performed within a certain time. In time-based ARPs, the service carried out on an edge must be completed within a specific time. If the service time cannot exceed this threshold, it is considered a hard time. When the service is allowed to be performed later than this time limit at the expense of a greater cost, it is called a soft time [?]. Several studies consider specific real-time-based applications of ARPs, such as mobile mapping [?], city courier and scheduling problems [?], winter road maintenance [?], railroad maintenance [?], snow removal [?], newspaper delivery, parcel delivery, and furniture delivery.

In 2026, Zhang et al. [?] proposed a collaborative home healthcare routing problem with time windows, where several collaborators share clients and jointly plan routes to reduce operational costs and gain a competitive advantage. They also proposed an arc-based model to formulate the problem and an effective branch-and-price algorithm, which integrates a column generation algorithm for solving the master problem and a tailored labeling algorithm to solve the pricing problems. In 2026, Li et al. [?], proposed a multi-depot collaborative electric vehicle routing problem with a heterogeneous fleet. They formulated a mixed integer programming model for the problem and developed a Q-learning enhanced variable neighborhood search algorithm, which improves search performance through the introduction of a Q-Learning mechanism and four newly designed neighborhood structures.

To the best of our knowledge, no study has been conducted on profitable arc routing problems considering overtime work and coalition satisfaction in the carrier collaboration framework. To fill this gap, this article proposes a bi-objective model for collaborative uncapacitated arc routing problem including coalition satisfaction in transportation sector. This model also is designed in such a way that a fair distribution of satisfaction among members is achieved.

3 Problem Definition

In this section, before describing our model, a motivating example is explained in 3.1. The motivation for proposing the B-CUARPS is an application described below. Then, the proposed model is formally described and formulated in 3.2.

3.1 Description and motivation

Fig.**??** shows regions 2 and 10 of Tehran. Two private companies (or carriers) distribute small packages and letters to home customers and there are a large number of customers on each street (arc). Each company has a vehicle and a number of assigned customers (per day) of two types; the first type are the ones that must be served by the company itself (required customers), or the ones that can be served by other carriers or left unserved [**?**], due to a low profitability or a low level of geographical synergies with other customers, (shared customers). Every day, each carrier's vehicle receives the postal packages that it has to deliver to the customers from the postal center. And then delivers the packages to the customers and returns to the center. The depots of carriers 1 and 2 are in nodes 10 and 34, respectively, which are shown with red solid circles. The required and shared arcs of the carriers are marked in red and green, respectively, which are solid arcs for carrier 1 and dashed arcs for carrier 2. Also the black arcs could be traversed without requiring service from which no revenue is collected. Associated with each arc, a triple including traversing time (minute), traversing cost and revenue (both multiplied by 10,000 Iranian Rial) collected from serving to each customer are written on that arc which are determined based on papers [**?**] and [**?**]. An optimization problem should be solved to find the corresponding routes for carriers.

If only profit maximization is considered (see the optimal route in

Fig.**??**), the vehicle of carrier 1 travels the route in 680 minutes and gain a profit of 1230 and these for vehicle of carrier 2 are 946 minutes 1530. The coalition's total profit is 2760. As it can be seen in Figure **??**, arcs (13,19), (16,17), (18,17), (21,22) and (23,24), of Carrier 1 do not receive service and due to dissatisfaction, the coalition may lose some (in large scale collaborative instances, even a large number) of its customers. In addition, the solution should not put too much pressure on the employees of some carriers and provide mental and physical health and work efficiency for the workforce for the next few days. Therefore, a fair distribution of satisfaction among coalition members should also be investigated. Furthermore, from the environmental perspective, we should consider greenhouse gas emissions reduction, while routing the vehicles. It can be either through adopting alternative fuel vehicles (greener transport options) or with existing conventional fossil fuel vehicles in fleets [**?**]. The less activity of the coalition will bring more benefits to the environment. Although using electronic vehicles contribute to less pollution and fuel consumption, nowadays they may not be available in many developing and underdeveloped countries. Even using these vehicles, extending their working hours can lead to higher taxes, noise, accidents, damage to the existing infrastructures (roads, bridges, etc.) and other possible environmental issues. Moreover, considering the high workload of some carriers, a part of their service time may occur during pick hours, which leads to traffic congestion and service delays. Extending their working hours as overtime may worsen these problems.

To tackle these issues, besides maximizing the total profit, a satisfaction objective is considered aiming at increasing the number of customers served and reducing the activity time of carriers. If we maximize both objectives simultaneously, a set of solutions will be achieved and among which, the decision maker can decide. For this example, two solutions obtained, one of which was already described. In the second solution, the shared arc (19,18) in the first solution is replaced by the arcs (13,19), (16,17) and (18,17). In this case, the vehicle of carrier 1 travels its route in 602 minutes and gain a profit of 1180, while the corresponding routes for vehicle of carrier 2 are 946 minutes 1530 and the coalition profit is 2710. Therefore, although the total profit has decreased slightly, but instead, more customers are served in less time. Note that, the set of

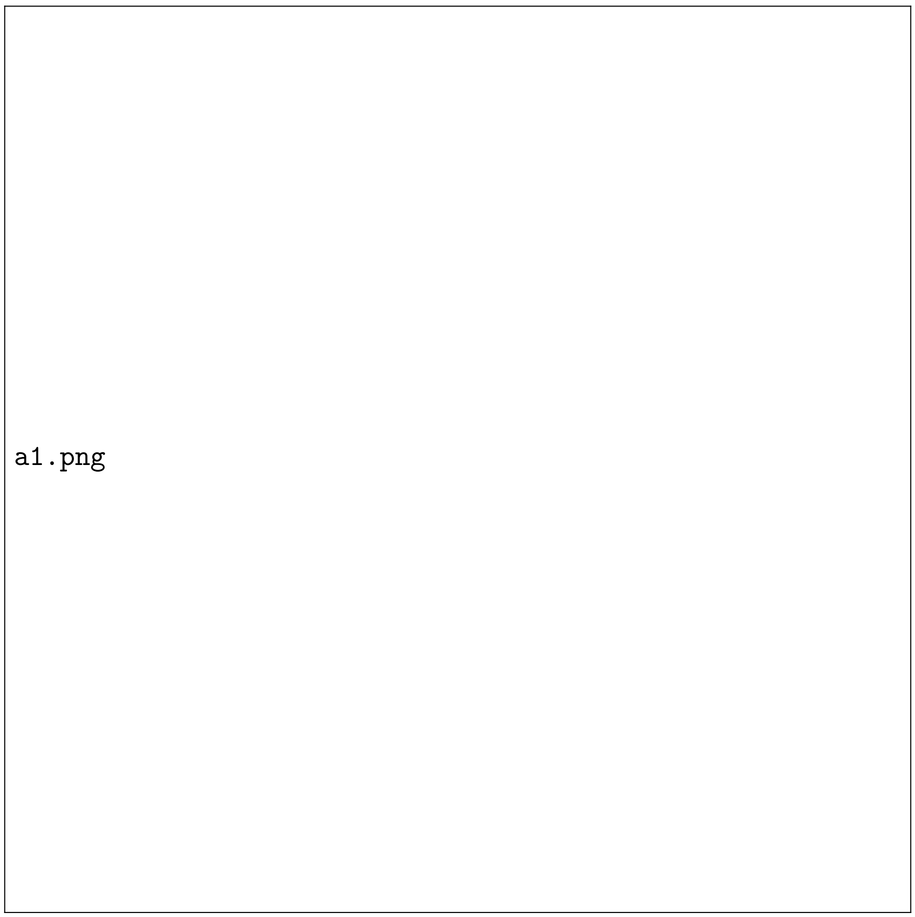

Figure 1: The graph of motivating example.

obtained solutions when considering the two objective functions includes the optimal solution(s) of solely considering the maximization of total profit. Hence, a more comprehensive set of solutions is achieved among which the coalition can choose.

a2.png

Figure 2: The optimal path for each carrier when maximizing the total profit (the right for carrier1 and the left for carrier2).

3.2 Problem definition and new formulation

Our problem formulation is a variant of [**?**]. A set of carriers, each with one depot and one uncapacitaed vehicle are considered. The network topology is represented by a strongly connected directed graph $G = (V, A)$ with vertex set V (including the depots of carriers) and arc set A whose arcs contain customers. Thus, the terms demand arcs, which will be defined in the following, and customers are interchangeably used.

In the following, the collaboration protocol is described which works as a guideline for carriers to cooperate under the supervision of a third-party central decision-maker. A carrier is associated with a vehicle, a depot, and a route, and it is assumed that the route starts and ends at the same depot. Each carrier aims at finding one route, regarding the collaboration scheme, so that both the total profit of the coalition and the coalition satisfaction are maximized. Each customer is assigned to a particular carrier.

The customers of each carrier are divided into two categories: required and shared. Required customers must be served by their assigned carrier due to contractual or strategic reasons, such as high profitability or confidentiality. Shared customers, on the other hand, can be served by any carrier or not served at all, usually because they are less profitable or don't align well geographically. While each customer is assigned to one carrier, multiple customers can be assigned to the same carrier. All the required and shared customers are called demand customers. The required and shared sets of the customers are denoted by R and , S the set of demand arcs $D = R \cup S \subseteq A$. For each arc $a \in A$, respectively, and their union constitutes each time this arc is traversed, a cost C_a is charged regardless of whetherais a demand arc or not. Each demand arc receives service when it is traversed by its assigned or another vehicle. A demand arc $a \in D$ which is served by its assigned carrier pays an amount of money r_a to the carrier. Moreover, for the case a shared arc $a \in S$ is served by a carrier different from the assigned one, part of the collected revenue of the assigned carrier which is a non-negative side payment $g_a (\leq r_a)$ is shared with the carrier serving the customer. It should be noted that, each arc can receive service at most once and by only one carrier. Thus, if a carrier is selected to serve an arc, the revenue is collected only once by that carrier (when the carrier traverses the arc more than one time). No penalty is considered for the shared customers which are unserved since our model is profit-based and some private companies establish a coalition to make as much profit as possible from a set of customers. However, due to time limitation and higher serving costs than their profitability, it is impossible/unreasonable to serve some of the customers. Note that all routes must start and end at the depot of their assigned carrier. The B-CUARPS goals are to maximize the total profit of the coalition of carriers and coalition satisfaction.

3.2.1 Carriers' overtime activity

In the existing problem formulations in the literature, each carrier is supposed to work at most for certain amount of time, T^U, each day. This is typically expressed as a hard constraint on carrier working time. However, in real world situations, carriers are often open to working extra hours if there is enough benefit that can justify the extra costs (e.g., staff payment, vehicle related cost). Indeed, overtime is often better than buying or renting extra vehicles and hiring new staff to operate them. Therefore, we propose allowing the carriers to work for at most T^O overtime hours. We denote the related overtime cost by φ^l, for carrier l. In other words, there is a soft constraint (T^U) and hard constraint ($T^U + T^O$) on each carrier working time. Here, φ^l denotes the amount of money paid to carrier l for each extra working hour.

3.2.2 Proposed formulation

Table **??** summarizes the notations used in this paper. There are two sets of decision variables in our formulation. For each arc $a \in A$, let x_a^l be the number of times vehicle I traverses a and

$$y_a^l = \begin{cases} 1 & \text{if } a \text{ is served by vehicle } l \\ 0 & \text{otherwise} \end{cases}$$

The two objective functions, as the total profit of the coalition and coalition satisfaction will be explained in the two following subsections.

Table 1: Parameters and Variable

Term	Definition	Term
$G = (V, A)$	Directed graph with vertex set V and arc set A	c_a
L, J	Set of carriers (or vehicles), $L = \{1, 2, ..., l\}$	g_a
$I^l = L \setminus \{l\}$	All carriers except for carrier l	T^U
V^l	The depot of carrier ^{l}	T^o
R^l	Required arcs of carrier ^{l}	t_a^d
S^l	Shared arcs of carrier ^{l}	t_a^S
D^l	Demand arcs of carrier ^{l}, $^{D^l=R^l\cup S^l}$ Note that $^{R^l\cap S^l=\emptyset}$	φ^l
R	Total required arcs,$R = \bigcup_{l\in L} R^l$	$\gamma(F)$
S	Total shared arcs, $S = \bigcup_{l\in L}$	y_a^l
D	Total demand arcs, $D = \bigcup_{l\in L} D^l or D = R \cup S$	x_a^l
r_a	Revenue collected from serving arc $a \in D$	τ^l

3.2.2.1 Total profit of the coalition

The individual profit of a carrier is the subtraction of the related costs from the revenue of that carrier, which will be described in the following. The first objective function is the total profit gained by the coalition as the sum of individual profits of all carriers.

Carriers' revenue; For carrier l, the total revenue is:

$$\sum_{a\in D^l} r_a y_a^l + \sum_{a\in S^l}\left[(r_a - g_a)\sum_{i\in I^l} y_a^i\right] + \sum_{i\in I^l}\sum_{a\in S^i} g_a y_a^l \qquad (1)$$

Where the first term is the profit from serving the carrier's assigned arcs, the second term is the revenue from other carriers serving shared customers of carrier l , and the last term represents the total side payments collected by carrier l from other carriers.

In our model, there are mainly two types of costs for a carrier l related to traveling cost and the overtime cost.

Carriers' traveling cost; the total traveling (or traversing) cost of the route of carrier l equals to $\sum_{a\in A} c_a x_a^l$.

Carriers' overtime cost; considering t_a^d and t_a^s as the traversing and service time of a typical arc a, the following equation denotes the completion time for the route of carrier l in hours.

$$T^l = \sum_{a \in A} t_a^d x_a^l + \sum_{a \in R^l \cup S} t_a^s y_a^l \tag{2}$$

Using (**??**), the following penalty term is proposed to penalize a carrier with more than T^U working hours:

$$\max\left\{0,\ \left(T^l - T^U\right)\right\} \times \varphi^l \tag{3}$$

In other words, if its completion time T^l exceeds T^U, i.e. $T^l > T^U$, carrier l is charged $\left(T^l - T^U\right) \times \varphi^l$.

The first objective; The total profit for carrier l is obtained by subtracting the cost from the revenue as follows:

$$\begin{aligned} P^l = \sum_{a \in E^l} r_a y_a^l + \sum_{a \in E^l} \left[(r_a - g_a) \sum_{i \in I} y_a^i \right] + \sum_{i \in I^l} \sum_{a \in S^l} g_a y_a^l \\ - \sum_{a \in A} c_a x_a^l - \max\left\{0, \left(T^l - T^U\right)\right\} \times \varphi^l \end{aligned} \tag{4}$$

The first objective function is formulated as the sum of individual profits:

$$f_1 = \max \sum_{l \in L} P^l \tag{5}$$

3.2.2.2 Coalition satisfaction

If the coalition's total profit is considered as the only criterion for the efficiency of the system, the collaboration may lead to serving the customers with higher profits or the customers of certain carriers. By doing so, the coalition will bias towards particular carriers. This might result in customer dissatisfaction with some carriers that are mainly working to serve other carriers' customers while their assigned shared customers are left unserved. As a consequence, the continuous customer dissatisfaction may lead the corresponding carriers to reconsider their commitment to the coalition. Therefore, in addition to the profitability of carriers, the quality of service should also be considered in the routing problem. From a shared customer's point of view, receiving service by any of the carriers in the coalition is desirable. In addition, for carriers, it is more desirable to complete their service (or route) no later than the normal working time, i.e. within T^U working hours.

In our model, two types of satisfactions are defined and combined to form the coalition satisfaction as follows:

Customer serving satisfaction; The more shared customers of a carrier are served (by any carrier) the more satisfaction it would receive from the customers. Hence, we consider the total number of served shared customers as the first type of satisfaction metric. For carrier l, a shared customer $a \in S^l$ is served if $\sum_{i \in I} y_a^i = 1$.

Therefore, $Q_1^l = \sum_{a \in s^l} \sum_{i \in I_a} y_i^a$ represents the number of served shared customers of carrier l.

Timely delivery satisfaction; Another satisfaction metric can be defined to reflect the timely delivery of the services. For establishing an environmentally friendly coalition or in case of imposing environmental policies by the government, the carriers would be expected to complete their services within the regular rather than extended hours (to reduce traffic congestion, noise, pollution, fuel consumption, accidents, damage to the existing infrastructures such as roads, bridges, etc.)

Therefore, we define:

$$Q_2^l = 1 - \max\left\{0, \frac{T^l - T^U}{T^o}\right\} \tag{6}$$

This metric is one if carrier l delivers the service within regular hours ($T^l \leq T^U$) and the metric decreases proportionally as the service is delayed.

In this step, using the two aforementioned types, another satisfaction concept is presented which will be used later both in our coalition satisfaction formulation and its evaluation in numerical experiments.

Carrier satisfactions; The total satisfaction metric for carrier l can be obtained by combining Q_1^l and Q_2^l as follows:

$$Q^l = \frac{1}{|s^l|} Q_1^l + Q_2^l \tag{7}$$

$Q_1^l$is multiplied by $\frac{1}{|S^l|}$for normalization purposes. $Q_1^l \in [0, S^l]$ but $Q_2^l \in [0, 1]$, and thus, they need to be normalized properly before combining them.

It is worth mentioning that one can also assign different weights to the two mentioned satisfaction types, i.e. $Q^l = w_1 Q_1^l + w_2 Q_2^l$, in order to balance the priorities of them.

The second objective; The coalition satisfaction can be defined through combining the introduced satisfactions concepts usingL_p-Norm. We propose combining L_1-Norm and L_∞-Norm representing the average and minimum satisfaction of carriers, respectively.

Using L_1-Norm, we define:

$$\|Q\|_1 = \frac{1}{|S|} \sum_{i \in L} Q_1^l + \frac{1}{|L|} \sum_{i \in L} Q_2^l = \frac{1}{|S|} \sum_{i \in L} \sum_{y \in S^l} y_a^l + \frac{1}{|L|} \sum_{i \in L} \left(1 - \max \left\{ 0, \frac{T^l - T^U}{T^O} \right\} \right) \tag{8}$$

since $S = \bigcup_{i \in L} S^l$and $S^l \cap S^h = \emptyset$ for all $l, h \in L$ with $l \neq h$, we have $|S| = \sum_{i \in L} |S^l|$. Hence, $\frac{1}{|S|} \sum_{i \in L} Q_1^l$ and $\frac{1}{|L|} \sum_{i \in L} Q_2^l$ are real numbers in $[0, 1]$.

L_∞-Norm promotes the improvement of the minimum satisfaction, resulting in a well-distributed satisfaction among all the carriers. In other words, L_∞-Norm improves the worst-case satisfaction.

$$\|Q\|_{-\infty} = \min_{l \in L} Q^l = \min_{i \in L} \left(\frac{1}{|S^l|} {Q_1}^l + {Q_2}^l \right) = \min_{l \in L} \left(\frac{1}{|S^l|} \sum_{a \in S^l} \sum_{j \in L} y_a^i + 1 - \max \left\{ 0, \frac{T^l - T^U}{T^O} \right\} \right) \tag{9}$$

Remark: Equation (**??**) is a weighted $L_1 - Norm$ (therefore a mathematical norm) with respect to Q, however, (**??**) is not a genuine mathematical norm and does not satisfy the properties of the norm. $\|x\|_{-\infty}$ is defined as $\|x\|_{-\infty} = \lim_{p \to -\infty} (\|x\|_p)$ and can be considered a generalized version of the L_p- Norm.

Combining (**??**) and (**??**), the second objective function is defined as (**??**):

$$Q_1 = \|Q\|_{-\infty} + \varepsilon\|Q\|_1 = \min_{l \in L}\left(\frac{1}{|S^l|}\sum_{a \in S^l}\sum_{i \in L} y_a^i + 1 - \max\left\{0, \frac{T^l - T^U}{T^O}\right\}\right)$$
$$+ \varepsilon\left[\frac{1}{|S|}\sum_{l \in L}\sum_{a \in S^l}\sum_{i \in L} y_a^i + \frac{1}{|L|}\sum_{i \in L}\left(1 - \max\left\{0, \frac{T^l - T^U}{T^O}\right\}\right)\right] \quad (10)$$

3.2.2.3 The significance of the defined objectives

The first objective often revolves around extending working hours, primarily driven by a cost-benefit analysis aimed at maximizing profits for carriers and the coalition. However, as exemplified in the motivating scenario, this singular focus may disregard crucial factors such as environmental sustainability and customer satisfaction. Simply prolonging working hours not only exacerbates environmental challenges but also fails to guarantee an increase in the shared customer base among carriers. Instead, it tends to favor high-profit customers, leading to an unfair distribution and dissatisfaction among unserved customers, who may constitute a significant portion of the clientele. On the contrary, the second objective strives to strike a balance by addressing environmental concerns and prioritizing customer satisfaction. This is achieved through a combination of reducing overtime hours and broadening the scope of served customers, ultimately fostering a more equitable and sustainable service model.

3.2.2.4 Linearization of the objective functions

The objective functions (**??**) and (**??**) are not linear and need to be linearized before solving the problem using a linear solver, such as CPLEX.

For carrier l, we define the following variable:

$$\delta_l := \max\{0, T^l - T^U\}, \quad l \in L \quad (11)$$

Which is equivalent to adding the following set of constraints to the model:

$$\delta_l \geq 0, \quad T^l - \delta_l \leq T^U, \quad l \in L \tag{12}$$

Using (**??**) and remark 3.1 in [**?**], the first objective function is linearized as:

$$f_1 = \max \sum_{l\in L} (\sum_{a\in D^l} r_a y_a^l + \sum_{a\in S^l} r_a \sum_{i\in I^l} y_a^i - \sum_{a\in A} c_a x_a^l - \delta_l \times \varphi^l) \tag{13}$$

Again using (**??**), we have:

$$Q_2 = \|Q\|_{-\infty} = \min_{l\in L} \left(\frac{1}{|S^l|} \sum_{a\in S^l} \sum_{i\in L} y_a^i + 1 - \frac{\delta_l}{T^o} \right) \tag{14}$$

and

$$Q_3 = \|Q\|_1 = \frac{1}{|S|} \sum_{l\in L} \sum_{a\in S^l} \sum_{i\in L} y_a^i + \frac{1}{|L| T^o} \sum_{l\in L} \delta_l + 1 \tag{15}$$

To linearize (**??**), we define the variable

$$z := \min_{l\in L} \left(\frac{1}{|S^l|} \sum_{a\in S^l} \sum_{i\in L} y_a^i - \frac{\delta_l}{T^0} + 1 \right) \tag{16}$$

We use the subsequent sets of constraints:

$$\begin{aligned} & z \geq 0 \\ & z \leq \frac{1}{|S^l|} \sum_{a\in S^l} \sum_{i\in L} y_a^i - \frac{\delta_l}{T^o} + 1, \quad l \in L \end{aligned} \tag{17}$$

Finally, the second objective is:

$$\begin{aligned} f_2 = \max & \left(z + \varepsilon \left[\frac{1}{|S|} \sum_{l\in L} \sum_{a\in S^l} \sum_{i\in L} y_a^i - \frac{1}{|L| T^o} \sum_{l\in L} \delta_l + 1 \right] \right) \\ & = \max \left(\|Q\|_{-\infty} + \varepsilon \|Q\|_1 \right) \end{aligned} \tag{18}$$

3.2.3 B-CUARPS: The integer linear model

This section describes the proposed bi-objective linear integer model.

Using (**??**) and (**??**), the objective function is:

$$\max \left\{ \sum_{l \in L} P^l, \|Q\|_{-\infty} + \varepsilon \|Q\|_1 \right\} \tag{19}$$

With the following set of linear constraints:

$$\sum_{a \in R^l \cup S} t_a^s y_a^l + \sum_{a \in A} t_a^d x_a^l - \delta_l \leq T^U, \qquad l \in L \tag{20}$$

$$z - \frac{1}{|S^l|} \sum_{a \in S^l} \sum_{i \in L} y_a^i + \frac{\delta_l}{T^o} \leq 1, \qquad l \in L \tag{21}$$

$$z \geq 0, \qquad 0 \leq \delta_l \leq T^o, \quad l \in L \tag{17}$$

Note that δ_l represents the overtime of carrier l that should be upper bounded by T^o.

The following sets of constraints, along with (**??**)-(**??**), model the collaboration agreement among carriers:

$$\sum_{i|(u,i)\in A} x^l_{ui} = \sum_{i|(i,u)\in A} x^l_{iu}, \qquad l \in L, \quad u \in V, \tag{23}$$

$$\sum_{i|(v^l,i)\in A} x^l_{v^l i} \geq 1, \qquad l \in L, \quad R^l \neq \emptyset, \tag{24}$$

$$\sum_{i\in(v^l,i)\in A} x^l_{v^l i} \geq \frac{1}{|S|}\sum_{a\in S} y^l_a, \qquad l \in L, \quad R^l = \emptyset, \tag{25}$$

$$\sum_{(i,j)\in A, i\in F, j\notin F} x^l_{ij} \geq y^l_a, \qquad F \subseteq V \setminus \{v^l\}, \quad a \in \gamma(F), \tag{26}$$

$$y^l_a = 1, \qquad a \in R^l, \quad l \in L, \tag{27}$$

$$\sum_{l\in L} y^l_a \leq 1, \qquad a \in D, \tag{28}$$

$$y^l_a \leq x^l_a, \qquad a \in A, \quad l \in L, \tag{29}$$

$$P^l \geq \tau^l, \qquad l \in L, \tag{30}$$

$$x^l_a \in \mathbb{Z}^+, \qquad a \in A, \quad l \in L, \tag{31}$$

$$y^l_a \in \{0,1\}, \qquad a \in D, \quad l \in L. \tag{32}$$

Constraints (**??**) ensure that the number of incoming arcs into each vertex must be equal to the number of outgoing arcs. According to constraints (**??**) and (**??**), the routes of carriers must start from their depots. While (**??**) force a route to any carrier $l \in L$ with non-empty required set, for any carrier with empty required set,

Constraints (**??**) force a route only if it serves some shared arc. The connectivity of the route of each carrier is guaranteed by constraints (**??**). For any subset $F \subseteq V \setminus \{v^l\}$,if an arc a, with both vertices in F, i.e. $a \in \gamma(F)$, is served, then some arc $b \in \{(i,j) \in A \mid i \in F, j \notin F\}$ must be traversed at least once. Thus, the route of each carrier is connected because if the carrier serves some arc from a subset of arcs not including the depot then it must leave the subset. However, note that even with holding constraints (**??**), subtours containing no served arc may be created. Since such subtours make no profit, they will not exist in any optimal solution. The set of constraints (**??**)–(**??**) also ensure that the route of each carrier ends at its depot. Constraints

(**??**) force carrier l to serve all of its assigned required arcs, whereas inequalities (**??**) ensure that each shared arc is served by at most one carrier. Finally, (**??**) ensures that each arc which is served by a given carrier is traversed by that carrier. Constraints (**??**) guarantee that the profit gained by carrier l in the collaboration scheme will not be less than that in the non-collaboration setting which is shown here by τ^l. (**??**) and (**??**) represents the Integrality constraints.

Now, using the first and second objective functions,f_1 (**??**) and f_2(**??**), and the feasible set Ψ(**??**)–(**??**) and assuming that η represents the vector of variables, we formulate our bi-objective model as follows:

$$\begin{aligned} \max f(\eta) &= (f_1(\eta), f_2(\eta)) \\ \eta &\in \Psi \end{aligned} \tag{32}$$

4 Solution Method

In this section, we briefly describe an algorithm named *augmented ε-constraint algorithm* [**?**] which we employ to solve our proposed bi-objective mixed integer problem (see Section 3.2.3). The main goal of the algorithm is to generate a set of potentially attractive Pareto optimal solutions including different trade-offs between two objective functions (i.e., total profit f_1 and CO_2 emissions f_2). This algorithm generates a set of Pareto candidates by solving a sequence of single-objective problems (steps 1-3) which are then used in our application and we solve them using a branch-and-cut algorithm in [**?**] implemented in CPLEX 12.6 with C# language.

Augmented ε-constraint Algorithm

The main idea of the augmented ε-constraint algorithm [**?**] is to first find a lower bound ($\varepsilon_{\min}$) and upper bound ($\varepsilon_{\max}$) for the second objective function f_2, and then generate different Pareto points by maximizing f_1 and constraining f_2 to different values in the interval $[\varepsilon_{\min}, \varepsilon_{\max}]$.

The steps of the augmented ε-constraint algorithm are as follows:

Input: Objective functions f_1 and f_2, solution space Ψ, number of grid points NGP, parameter eps (a small number to incentivize higher objective value for f_2 beyond the specified constraint);

Step 1: f_1 is maximized on Ψ;

Step 2: The lexicographic phase is implemented, i.e. by maximizing f_2 on the optimum solution of f_1. The optimal solution will be the lower bound for f_2, denoted by $\varepsilon_{2,\min}$;

Step 3: f_2 is maximized on Ψ. The optimal solution will be the upper bound for f_2, denoted by $\varepsilon_{2,\max}$;

Step 4: Set $r = \varepsilon_{2,\max} - \varepsilon_{2,\min}$ and $i = 1$;

Step 5: Set $\varepsilon_2 = \varepsilon_{2,\min} + \frac{i \times r}{NGP-1}$ and maximize the augmented objective function $f_1 + eps \times s_2$ on the space $\Psi \cup \{f_2 - s_2 = \varepsilon_2,\ s_2 \geq 0\}$ and **return** the optimal solution;

Step 6: Check **if** $i = NGP$, stop; **else**, set $i = i + 1$ and go to Step 5;

Output: NGP efficient solutions for the B-CUARPS (after removing probably identical obtained solutions).

Readers are referred to [**?**] for more information about this algorithm.

5 Numerical Results

In this section, we first clarify and motivate our work using three simple examples. Later, we evaluate our technique on a set of public datasets provided by different groups ([**?**], [**?**], [**?**], [**?**]) and also being used in [**?**].

5.1 Simple motivating examples

Example 1 (Overtime Impacts). Fig.**??** shows a small graph with four vertices and two carriers whose depots are at vertices 1 and 2, respectively. The arcs are partitioned as follows:

$R^1 = \{(1,2)\}, R^2 = \{(2,1)\}$,
$S^1 = \{(1,3),(3,2),(3,4)\}, S^2 = \{(3,1),(2,3),(4,3)\}$

Each arc in Fig.**??** is linked to four numerical values, denoting the traversing cost and serving profit (both multiplied by 10,000 Iranian Rial), as well as the traversing time and service time (both measured in minutes), respectively.

In this example, we set the cost of working overtime for both carriers to be 2 units per hour $\left(\varphi^1 = \varphi^2 = \frac{3}{8} \times \frac{40}{100} = 0.15\right)$ and the side payment for each shared arc a equals to half the profit of that arc ($g_a = 0.5r_a$).

a3.png

Figure 3: The instance graph.

We assume that the normal working hours is 8 hours or equivalently 480 min, i.e. $T^U = 480$ min. In order to investigate the impact of allowing overtime activity on the carriers' profit, three scenarios are implemented. We consider no overtime ($T^\circ = 0$ min), 2-hour overtime ($T^\circ = 120$ min) and 4-hour overtime ($T^\circ = 240$ min) scenarios in this study.

We use the following notations for arcs in the tables. The only traversed arcs are depicted by "$\rightarrow$" and the served arcs by "$\mapsto$".

Table **??** and **??**, and Fig.**??** demonstrate how working overtime can improve the total profit of the coalition in non-collaboration and collaboration settings, respectively.

Table **??** shows the routes' time lengths (T^l) and profits (P) for each carrier in each scenario in the non-collaboration setting. It is easy to see that the total profit is risen from 6 to 10 by allowing overtime hours (See Fig.**??**). The total number of served arcs are also increased from 3 to 7, moving towards the right of Table **??**.

Table **??** shows the optimal solutions for the three scenarios, this time in collaboration setting. The data shows an increase from 12 to 14.7 in the total profit (See Fig.**??**) and from 6 to 8 in the total number of served arcs and as a result more customers are satisfied with services. Furthermore, comparing Table **??** and **??**, one can see the optimal total profit in the first scenario for two tables are 6 and 12, in the second scenario are 8 and 13.7 and in the final scenario are 10 and 14.7, respectively (See Fig.**??**). It is induced that the incorporation of overtime working hours in collaboration setting can reap better results as opposed to that in non-collaboration case which indicates the benefit of collaboration among carriers.

a4.png

Figure 4: Total profit with and without collaboration for the three overtime scenarios

Table 2: The optimal solution for non-collaboration setting when profit is maximized.

Carrier	Non-collaboration								
	$T^0 = 0$ min			$T^0 = 120$ min			$T^0 = 240$ min		
	Path	T^I	P	Path	T^I	P	Path	T^I	P
Carrier 1	$1 \mapsto 2 \to 3 \mapsto 2 \to 1$	420	3	$1 \mapsto 2 \to 3 \mapsto 4 \to 3 \mapsto 2 \to 1$	560	4	$1 \mapsto 2 \to 1 \mapsto 3 \mapsto 4 \to 3 \mapsto 2 \to 1$	720	6
Carrier 2	$2 \mapsto 1 \to 2$	220	3	$2 \mapsto 1 \to 2 \to 3 \mapsto 1 \to 2$	510	4	$2 \mapsto 1 \to 2 \mapsto 3 \mapsto 1 \to 2$	570	4
Total Profit			6			8			10

The following example is to investigate the importance of considering coalition satisfaction criterion when solving a collaborative routing problem. This is done by considering only profit, as the objective function, and then analyzing the solutions obtained.

Example 2 (Ignoring coalition Satisfaction). Fig.**??** depicts a small graph with seven vertices $\{1, 2, 3, 4, 5, 6, 7\}$, twenty-seven arcs, and three

Table 3: The optimal solution for collaboration setting when profit is maximized.

Carrier	Collaboration								
	$T^0 = 0$ min			$T^0 = 120$ min			$T^0 = 240$ min		
	Path	T^I	P	Path	T^I	P	Path	T^I	P
Carrier 1	$1 \mapsto 2 \mapsto 3 \mapsto 1$	450	6	$1 \mapsto 2 \to 3 \mapsto 4 \mapsto 3 \mapsto 1$	580	5.7	$1 \mapsto 2 \mapsto 3 \mapsto 1$	450	10
Carrier 2	$2 \mapsto 1 \mapsto 3 \mapsto 2$	420	6	$2 \mapsto 1 \mapsto 3 \mapsto 2$	420	8	$2 \mapsto 1 \mapsto 3 \mapsto 4 \mapsto 3 \mapsto 2$	610	4.7
Total Profit			12			13.7			14.7

carriers whose origins are at vertices 1, 2 and 3, respectively. The arcs are divided as follows:

$R^1 = \{(1,2),(1,4)\}, \quad R^2 = \{(2,1),(2,6)\}, \quad R^3 = \{(3,5),(3,7)\},$
$S^1 = \{(1,6),(1,7),(2,3),(3,1),(4,1),(4,7),(5,3),(5,4),(5,7),(6,2),$
$(6,4),(6,5)\},$
$S^2 = \{(6,3),(7,1),(7,6)\}, \quad S^3 = \{(1,3),(4,5),(5,1),(5,6),(7,3)\}.$

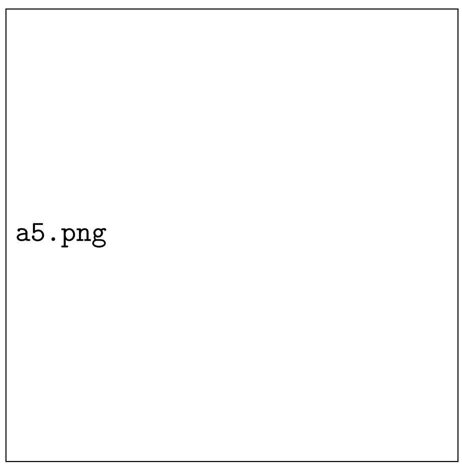

Figure 5: Instance graph.

We set $(\varphi^0 = \frac{34}{8} \times \frac{40}{100} = -x = -1.7)$, $(\varphi^1 = \frac{20}{8} \times \frac{40}{100} = -x = -1)$, $(\varphi^2 = \frac{24}{8} \times \frac{40}{100} = -x = -1.2)$ and 4-hour overtime working $(T^0 = 0.50 \times T^u = 0.50 \times 480 \text{ min} = 240 \text{ min})$.

Since only maximizing profit is considered in this example, the profit of individual carriers increased in the collaboration mode compared to the non-collaboration case (Table **??** and Fig.**??**). However, this increasing trend is not reflected in the number of shared arcs that are served,

which impacts the carrier satisfactions (see equation (**??**)). Although the total number of served arcs increases from 10 without collaboration to 15 with collaboration, the distribution of servicing among carriers is not well-balanced. Specifically, out of the 15 served customers, 10 are assigned to carrier1, while carrier2 and carrier3 each serve 2 arcs. Note that the variation percentages for carrier1 are calculated as
$(\frac{93.2-54.05}{54.05} \times 100 = +72.43)$, $\frac{0.846-0.509}{0.509} \times 100 = +66.23)$, $\frac{(11-4)}{4} \times 100 = +175.0)$, The same calculations are valid for carriers 2 and 3.

Table 4: Optimal results when profit is the only objective to be optimized.

carrier	Non-collaboration			Collaboration		
	Profit	Carrier Satisfaction	Served Shared Arcs	Profit (%)	Carrier Satisfaction (%)	Served Shared Arcs
Carrier 1	54.05	0.509	4	93.2 (+72.43)	0.846 (+66.21)	11(+175.00)
Carrier 2	20.33	1.333	3	30.5 (+50.02)	0.791 (-40.66)	2 (-33.33)
Carrier 3	31.8	1.35	3	37.59 (+18.21)	0.483 (-64.22)	2 (-33.33)

According to Table **??** and Fig.**??**, as a result of collaboration, the satisfaction for carrier1 has increased by 66.21%, while the satisfaction has dropped by 40.66% and 64.22% for carrier2 and 3, respectively. This, in turn, will lead the coalition to serve more profitable customers of certain carriers and ignore a potentially large part of the customer set. In the long term, the downside of this trend on the coalition will be revealed. To soften this phenomenon, the coalition satisfaction metric should also be taken into account in the optimization process and finding carriers' routes.

In the following experiment, we investigate the impact of overtime working hours on both the coalition's overall profit and the total number of shared arcs served, as well as on the profit and the number of shared arcs served for each individual carrier within the coalition (see Table **??**).

Table 5: The impact of overtime in collaboration setting.

Carrier	Collaboration								
	$T^0 = 0$ min			$T^0 = 120$ min			$T^0 = 240$ min		
	Profit	Shared Arcs	$Routes'TimeLength(T^r)$	Profit	Shared Arcs	$Routes'TimeLength(T^r)$	Profit	Shared Arcs	$Routes'TimeLength(T^r)$
Carrier1	53	6	480	84.3	9	540	93.2	11	720
Carrier2	35	2	460	28	1	600	30.5	2	690
Carrier3	25	1	480	29.8	1	540	37.59	2	700
Total	113	9	1420	142.1	11	1680	161.29	15	2110

According to the table, for carrier1, the augmentation of overtime

a6.png

Figure 6: The change (%) in profit and carrier satisfaction in collaboration setting.

hours correlates with a significant escalation in both profit and served shared arcs, soaring from 53 to 93.2 and from 6 to 11, respectively. Conversely, carrier3 experiences a boost solely in profit, climbing from 25 to 37.59, whereas an additional 120 minutes of overtime fails to elevate served shared arcs. In stark contrast, carrier2 demonstrates a contrasting pattern; the inclusion of 120 minutes of overtime precipitates declines in both profit (falling from 35 to 28) and served shared arcs (diminishing from 2 to 1). It's worth noting that even in this scenario, carrier2 benefits from the collaboration, as the profit (28) exceeds that in non-collaboration mode (20.33), with maximum overtime working (see Table 4). Consequently, while the adoption of overtime enhances the overall profit (ascending from 113 to 161.29) and total served shared arcs (rising from 9 to 15), aligning with the coalition's objectives, it may engender a reduction in profit and/or served shared arcs for certain carriers. These experiments motivated us to consider the two defined objective functions simultaneously, resulting in the generation of a set of Pareto solutions from which the decision maker can select (see example 3).

Example 3 (bi-objective model). We use the same data as Example 2 and employ the proposed bi-objective model (see Section 3.3) and generate 10 Pareto solutions using augmented ε-constraint algorithm presented in Section 4 (NGP = 10, $\varepsilon = 10^{-3}$) . Fig. **??** represents all Pareto candidates for non-collaboration and collaboration settings in coalition satisfaction–profit space. (certain points within each Pareto set may overlap due to their proximity in the figure).

In this figure, the extreme points (106.12, 1.436) and (79, 2.564) in the non-collaboration setting, and (161.33, 1.309) and (106, 2.813) in the collaboration setting of the bi-objective problem represent optimal solutions for maximizing profit and coalition satisfaction, respectively. Intermediate points exhibit a trade-off between coalition satisfaction and profit, with coalition satisfaction increasing at the expense of profit. This flexibility provides decision-makers with a range of options. Notably, the number of solutions obtained is customizable, enhancing adaptability. Profit intervals [79, 106.12] and [106, 161.33], and coalition satisfaction intervals [1.436, 2.564] and [1.309, 2.813] indicate higher levels of performance in the collaborative setting. In conclusion, the Pareto set characterized by collaboration dominates its counterpart without collaboration which emphasizes the importance and impacts of collaboration among carriers.

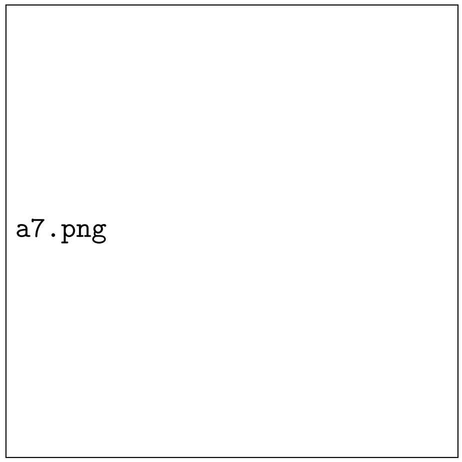

Figure 7: Pareto optimal solutions for B-CUARPS in non-collaboration and collaboration settings.

In Table **??**, we will conduct a more detailed analysis of the distribution of solutions, in the collaboration mode, depicted in Fig. **??**. Each row in the table corresponds to the information of a Pareto point from Fig **??**. The columns of the table for each carrier represent the number of shared arcs served (NSA), the percentage of these arcs in relation to the total shared arcs of the carrier (%NSA), route time (T), satisfaction (CS), and profit (P) of that carrier in the obtained solution. The last two columns of the table display the total profit and satisfaction of the coalition, which form the ordered pair of the related point in Fig. **??**.

In this example, transitioning from point 1 to point 10, the profit of the coalition decreases while the satisfaction increases. Upon examining the parameters associated with each carrier's satisfaction, it becomes evident that the reduction in route time is the primary reason for the increase in carrier satisfaction. Put differently, in this example and within the defined problem topology and time frame, prioritizing the satisfaction objective function tends to mitigate environmental consequences.

Table 6: The detailed information of the carriers for Example 3.

	Carrier 1					Carrier 2					Carrier 3						
	NSA	%NSA	T	#Cs	#P	NSA	%NSA	T	#Cs	#P	NSA	%NSA	T	#Cs	#P	T_{ot}	T_{otCs}
1	11	84.62	720	0.846	93.2	2	66.67	700	0.750	33.33	2	40	690	0.525	34.8	161.33	1.309
2	11	84.62	720	0.846	90.2	2	66.67	720	0.667	35	2	40	650	0.692	32.6	157.8	1.478
3	10	76.92	540	1.519	90.3	2	66.67	720	0.667	39	1	20	620	0.617	28.2	157.5	1.625
4	10	76.92	720	0.769	84.2	2	66.67	620	1.083	36.67	1	20	500	1.117	30.6	151.47	1.833
5	9	69.23	540	1.442	80.3	2	66.67	690	0.792	34.5	1	20	500	1.117	30.6	145.4	1.96
6	8	61.54	540	1.365	75.3	2	66.67	610	1.125	37.83	1	20	510	1.075	28.4	141.53	2.293
7	8	61.54	590	1.157	66.88	2	66.67	610	1.125	37.83	2	40	510	1.275	33.4	138.12	2.321
8	7	53.85	560	1.205	61.73	2	66.67	540	1.417	32	2	40	500	1.317	36.3	130.33	2.507
9	7	53.85	540	1.288	67.3	1	33.33	470	1.333	23	2	40	500	1.317	35.6	125.9	2.654
10	5	38.46	480	1.385	45	2	66.67	470	1.667	30	2	40	450	1.4	31	106	2.813

5.2 Experiments on public datasets

We use the CUARP benchmark instances [**?**] which is a collection of datasets from different groups. All the C#/CPLEX codes are run on a laptop with 8GB RAM and 1.19GHz Intel CPU. The missing data for our model are time (travel and service time) and the overtime cost for each carrier. We follow the idea presented in [**?**] to generate time data as follow:

1. The length of each arc a (in kilometers) is one tenth of the cost of that arc: $len_a = 0.1c_a$.

2. In physics, the average speed of a vehicle is equal to the distance traveled divided by the travel time. If we consider the average speed of the vehicle to be 35 km/h, the travel time of that arc (in minutes) is equal to: $t_a = \frac{60l_a}{35}$.

3. For each required arc, the number of customers on that arc is considered in the range $[1, 5]$: $q_a \in [1, 5]$.

4. For each required arc a, the service time is equal to: $q_a = t_a + 5q_a$, i.e. 5 minutes for servicing each customer.

Furthermore, for setting the overtime cost per hour for a carrier $l^{(\varphi^l)}$, we use the following strategy. The optimal profit gained by this carrier in the non-collaboration environment is divided by the total time length of that carrier's route and forty percent of it is considered as an overtime pay per hour for the carrier.

Table 7: Dataset for all experiments' instances.

Group Instances	Number of Instance	Number of Vertices	Number of Required Arcs					Number of Shared Arcs				
			2-Carrier		3-Carrier			2-Carrier		3-Carrier		
			$\lvert R^1\rvert$	$\lvert R^2\rvert$	$\lvert R^1\rvert$	$\lvert R^2\rvert$	$\lvert R^3\rvert$	$\lvert S^1\rvert$	$\lvert S^2\rvert$	$\lvert S^1\rvert$	$\lvert S^2\rvert$	$\lvert S^3\rvert$
A	2	90–102	48–50	23–34	46-90	11-22	17	48-50	23-35	41-46	12-23	17
D	36	16–100	5–67	7–50	2-61	6–40	1–29	5–67	8–50	2–62	6-40	2-29
G	36	16–100	0-29	0-32	0–23	0–24	0–14	1–29	1–33	1-24	1-24	0-14
P	24	7–50	0–48	2–46	1–43	1–42	1–22	1–48	3–47	2–44	1-42	1-23
R	20	20–50	10–57	5–56	6–53	3–45	3–28	10–58	6–56	6–54	3-45	4-45

Table 8: Number of instances for each combination of parameters and scenarios.

Number of Carriers	Num of Instances (Non-collaboration)	Num of Instances (Collaboration)		Metric			Scenario (overtime)	
		coalition satisfaction	Profit	Q^1	Q^2	Q^3	$T^O = 0.25T^U$	$T^O = 0.5T^U$
2	118	118	118	—	—	—	118	118
3	—	118	118	118	118	118	118	118

Table **??** summarizes the information related to instances grouped according to their features and sizes. These instances are generated with two and three depots. The number of group instances and the range of vertex number of each group instance are shown in the second and third columns, respectively. Columns $|R^l|$ and $|S^l|$ $(l = 1, 2, 3)$ represent the numbers of required and shared arcs of carriers one, two, and three, respectively. Table **??** summarizes the information about the values of the parameters used and the number of instances tested in each case, , resulting in 1416 instances in total.

In the following experiments, three scenarios including without overtime $(T^O = 0)$, using 25% of working time as overtime $(T^O = 0.25T^U)$ and 50% of working time as overtime $(T^O = 0.5T^U)$ are considered. To show the effectiveness of our proposed coalition satisfaction metric Q1 (10), it is compared with two other metrics Q2 (14), and Q3 (15), through 118 instances in the collaboration setting. Note that except experiment3 where different values of ε are considered, in all relevant experiments we use $\varepsilon = 1$. Moreover, all the following optimization problems are subject to the set of constraint (20) - (31).

Experiments 1 *(overtime impact on profit and coalition satisfaction)*: In the experiment that follows, we examine the influence of overtime on the Pareto set within our bi-objective model on the instances from dataset, considering three distinct overtime scenarios. Among all the results obtained, three main patterns were extracted and shown in Fig.**??**.

In Fig.**??** (a) and (b), it is evident that a rise in overtime leads to a decrease in coalition satisfaction, albeit with a simultaneous increase in overall profit. In this instance, augmenting overtime hours enhances the diversity of the Pareto set. This leaves the decision maker's hand open, offering them a plethora of options and expanding their choices. In Fig. **??** (c), it can be observed that, in general, as overtime hours increased, the level of satisfaction also increased, resulting in a better distributed Pareto set.

a8.png

Figure 8: Overtime impact on the Pareto set.

The similar analysis of the solutions as in Table 6 for Fig. **??** (a), for overtime 240 min is depicted in **??**. It is observed that point 4 represents a fair solution for this problem, considering both profits and carrier satisfaction. Generally, the bi-objective model offers the decision-maker a wide range of solutions distributed throughout the objective space.

The impact of overtime was also investigated on the three-carrier

Table 9: The detailed information of the carriers for Fig. 8(a).

Row	Carrier 1					Carrier 2					Tot.P	Tot.Cs
	NSA	%NSA	T	Cs	P	NSA	%NSA	T	Cs	P		
1	27	77.14	4326	1.694	21744.3442	9	56.25	5112	1.128	13674.34	35418.84	2.578
2	26	74.28	4407	1.644	21892.39	9	56.25	4984	1.189	13541.66	35407.05	2.639
3	25	7428	4407	1.644	21892.39	9	56.25	4984	1.189	13541.66	35407.05	2.639
4	25	71.42	4889	1.381	22686.16	11	68.75	4818	1.393	12338.7	35024.86	2.773
5	26	71.42	4899	1.381	22686.16	11	68.75	4818	1.393	12338.7	35024.86	2.773
6	25	71.42	4789	1.434	23535.06	9	56.25	4487	1.426	11665.54	35200.6	2.884
7	24	68.57	4622	1.484	21312.38	8	50.00	4260	1.471	13741.04	34992.37	2.983
8	22	68.57	4425	1.521	21312.38	8	50.00	4230	1.485	13559.9	34872.28	3.013
9	22	68.57	4410	1.529	19694.72	9	56.25	4275	1.527	12341.28	32036.01	3.067
10	22	62.86	4410	1.529	19694.72	9	56.25	4275	1.527	12341.28	32036.01	3.067

instance in Example 2 and the results were depicted in Fig.**??**. It is evident that overtime not only enhances the diversity of the Pareto set but also yields higher quality solutions compared to the scenario without overtime.

In [**?**], it is shown that collaboration increases the individual profit of each carrier compared to when they work independently (without collaboration). In experiment2, the influence of collaboration on the coalition satisfaction is analyzed.

Experiments 2 *(collaboration impact on coalition satisfaction)*:

Here, coalition satisfaction is maximized on the instances with two carriers in Table 7, considering the third scenario in two cases. The average figures for the two cases with and without collaboration are shown in columns $\#CS_c$ and $\#CS_{nc}$ of Table 10, respectively. In the column $\#CS_c$, in addition to the average coalition satisfaction, the growth percentage of the average coalition satisfaction in the collaborative mode compared to the non-collaborative mode $\left(100\frac{\#CS_c-\#CS_{nc}}{\#CS_{nc}}\right)$ is also shown, in parentheses. This table shows that collaboration has caused an average increase of 9.94% and 18.40% in the percentage of coalition satisfaction.

Experiments 3 *(validation of our coalition satisfaction metric)*: In the last experiment, to validate our proposed coalition satisfaction metric, i.e. $Q = \|Q_a\|_1 + \varepsilon\|Q_a\|_\infty$, the results of maximizing this metric over instances with three carriers are compared with those of two metrics $Q_2 = \|Q\|_{-\infty}$ and $Q_3 = \|Q\|_1$ represented in Table 11. Indeed, these metrics represent special cases of our metric with different ϵ values.

a9.png

Figure 9: Overtime impact on the pareto set on Example 2.

Table 10: (Experiment 2)Collaboration impact on coalition satisfaction

Group Instance	Coalition Satisfaction (non-collaboration)	Coalition Satisfaction (collaboration)(Increase %)
	$\#CS\_nc$	$\#CS\_c$
A	2.635	3.229 (+22.54)
D	2.432	2.885 (+18.63)
G	2.431	2.821 (+16.04)
P	2.347	2.606 (+11.03)
R	2.826	3.271 (+15.75)
Average	**2.534**	**2.962 (+16.89)**

Specifically, the (Q_2) metric corresponds to $\varepsilon = 0$, while the Q_3 metric corresponds to a large value for ε (i.e. $\varepsilon \to +\infty$). For each metric, there are three optimal paths obtained for each one of the samples of different groups. The box plot for the mean column for each of the metrics is plotted and shown in Fig.10. Because the goal of the third metric Q_3 is to maximize the total satisfaction (sum of all carriers' satisfaction), it achieves the best average satisfaction; however, it ignores the distribution of satisfaction among the carriers and the minimum satisfaction is lowest among all the metrics. Metric Q_2, on the other hand, only pays attention to the minimum satisfaction and ignores the average satisfaction. Our proposed metric Q_1 strikes a balance between these two metrics by paying attention to both average and minimum satisfaction and it achieves the minimum satisfaction almost as good as Q_2 but better average satisfaction. The box plot in Table 11 and Fig.10 shows this metric can be a more appropriate metric for measuring coalition satisfaction.

Table 11: Carrier satisfaction for different choices of metrics as the second objective function

Group	Metric 1: $Q_1 = \|\mathbf{Q}_L\| + \varepsilon\|\mathbf{Q}_L\|_\infty$			Metric 2: $Q_2 = \|\mathbf{Q}_L\|_\infty$			Metric 3: $Q_3 = \|\mathbf{Q}_L\|$		
	Carrier 1	Carrier 2	Carrier 3	Carrier 1	Carrier 2	Carrier 3	Carrier 1	Carrier 2	Carrier 3
A	1.320	1.327	1.310	1.324	1.310	1.323	1.173	1.352	1.683
D	1.347	1.440	1.491	1.477	1.478	1.451	1.289	1.764	1.491
G	1.356	1.109	1.020	1.141	1.036	1.218	1.398	0.998	1.143
P	0.925	1.066	1.061	0.925	1.019	0.943	0.767	1.302	1.067
R	0.858	1.036	0.869	0.885	0.889	0.889	0.517	1.144	1.496

On the computational side, the average CPU time required to solve small instances, such as groups with vertices less than 36 vertices, was less than 2 seconds, groups with vertices between 36 and 50 vertices was between 2s and 48s and group with more than 50 vertices required more

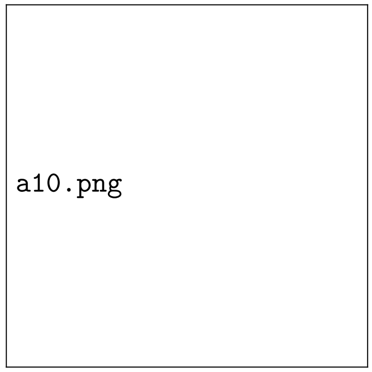

Figure 10: The distribution of the average and worst satisfactions among carriers for different choices of metrics as the second objective function.

computational effort and reached the CPU time of 273 seconds.

6 Conclusions

In this paper, we have introduced a bi-objective variant of the Collaborative Uncapacitated Arc Routing Problem (CUARP). Since the CUARP and its variant, t-CUARP in [12], only considers the profit of the coalition and the carriers involved, the coalition may tend to serve only the most profitable parts of the shared customer set. This will lead to dissatisfaction among less-considered carriers (and their customers), which in turn adversely affects the future of the collaboration in the long term. For this reason, we considered profit and coalition satisfaction simultaneously in our model. Using proper mathematics metrics, achieving fair distributed satisfaction is also investigated in the proposed model. Moreover, overtime working is allowed and integrated with the profit objective to increase the chance of profitability among carriers and the coalition itself. Considering profit and coalition satisfaction as two objective functions, an integer linear programming formulation with binary and integer variables has been proposed, and solved with an augmented ϵ-constraint algorithm. A set of benchmark instances were used to evaluate the model and algorithm and the results of computational experiments were presented and analyzed.

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