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Original Research Paper

(Weakly) (α, β)-Prime Hyperideals in Commutative Multiplicative Hyperrings

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Abstract. Let H be a commutative multiplicative hyperring and $\alpha, \beta \in \mathbb{Z}^+$. The purpose of this paper is to introduce an intermediate class between prime hyperideals and (α, β)-closed hyperideals called (α, β)-prime hyperideals. Moreover, we study the notion of weakly (α, β)-prime hyperideals as an extension of the (α, β)-prime hyperideals and a subclass of the weakly (α, β)-closed hyperideals. We say that a proper hyperideal P of H is (weakly) (α, β)-prime if ($0 \notin x^\alpha \circ y \subseteq P$) $x^\alpha \circ y \subseteq P$ for $x, y \in H$ implies $x^\beta \subseteq P$ or $y \in P$. A number of properties and results concerning them will be discussed.

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1 Introduction

Delving into the study of prime ideal generalizations has emerged as a profoundly intriguing and groundbreaking pursuit in the realm of commutative ring theory. In a recent study [19], Khashan and Celikel presented (α, β)-prime ideals which is an intermediate class between prime

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ideals and (α, β)-closed ideals. Let $\alpha, \beta \in \mathbb{Z}^+$. A proper ideal I of a commutative ring R refers to an (α, β)-prime ideal if for $x, y \in R$, $x^\alpha y \in I$ implies either $x^\beta \in I$ or $y \in I$. Moreover, the authors generalized this notion to weakly (α, β)-prime ideals in [20]. A proper ideal I of R is called a weakly (α, β)-prime ideal if for $x, y \in R$ with $0 \neq x^\alpha y \in I$, then either $x^\beta \in I$ or $y \in I$.

Several key concepts in modern algebra were expanded by extending their underlying structures to hyperstructures. In 1934 [21], the French mathematician F. Marty pioneered the notion of hyperstructures or multioperations, where an operation yields a set of values rather than a single value. Subsequently, numerous authors have contributed to the advancement of this novel area of modern algebra [7, 8, 9, 10, 11, 22, 24, 28]. An importan type of the algebraic hyperstructures called the multiplicative hyperring was introduced by Rota in 1982 [26]. In this hyperstructure, the multiplication is a hyperoperation and the addition is an operation. Multiplicative hyperrings are richly demonstrated and characterized in [2, 3, 4, 18, 16, 25, 27]. Dasgupta studied the prime and primary hyperideals in multiplicative hyperrings in [12]. The idea of (α, β)-closed hyperideals in a multiplicative hyperring was proposed in [5]. A proper hyperideal P of a multiplicative hyperring H is said to be (α, β)-closed if for $x \in H$ with $x^\alpha \subseteq P$, then $x^\beta \subseteq P$. Motivated from this notion, the aim of this research work is to introduce and study the notion of (α, β)-prime hyperideals in a commutative multiplicative hyperring. Several specific results are given to illustrate the structure of the new notion. We show that every (α, β)-prime hyperideal is an (α, β)-closed hyperideal but the converse need not to be hold in Example 3.4. We obtain that if P is an (α, β)-prime $\mathcal{C}$-hyperideal of H, then $rad(P) = \{x \in H \mid x^\beta \subseteq P\}$ in Theorem 3.10. We present a generalization of Prime Avoidance Theorem for (α, β)-prime hyperideals in a multiplicative hyperring in Theorem 3.22. Furthermore, we extend this notion to weakly (α, β)-prime hyperideals. We present some characterizations of weakly (α, β)-prime hyperideals on cartesian product of commutative multiplicative hyperrings.

The following diagram shows the place of (α, β)-prime and weakly (α, β)-prime hyperideals for all $\alpha, \beta \in \mathbb{Z}^+$:

$$
\begin{array}{ccccc}
 & & \text{primary} & & \\
 & \nearrow & \uparrow & & \\
\text{prime} & \longrightarrow & (\alpha,\beta)\text{-prime} & \longrightarrow & (\alpha,\beta)\text{-closed} \\
\downarrow & \searrow & \downarrow & \searrow & \downarrow \\
\text{w. prime} & \longrightarrow & \text{w. }(\alpha,\beta)\text{-prime} & \longrightarrow & \text{w. }(\alpha,\beta)\text{-closed} \\
 & \searrow & \downarrow & & \\
 & & \text{w. primary} & &
\end{array}
$$

2 Some Basic Definitions Concerning Multiplicative Hyperrings

In this section we give some basic definitions and results which we need to develop our paper. [14] A hyperoperation " $\circ$ " on non-empty set I is a mapping from $I \times I$ into $P^*(I)$ such that $P^*(I)$ is the family of all non-empty subsets of I. In this case, $(I, \circ)$ is called hypergroupoid. Let I_1, I_2 be two subsets of I and $x \in I$, then $I_1 \circ I_2 = \cup_{x_1 \in I_1, x_2 \in I_2} x_1 \circ x_2$, and $I_1 \circ x = I_1 \circ \{x\}$. This means that the hyperoperation " $\circ$ " on I can be extended to subsets of I. A hypergroupoid $(I, \circ)$ is called a semihypergroup if $\cup_{a \in y \circ z} x \circ a = \cup_{b \in x \circ y} b \circ z$ for all $x, y, z \in I$ which means $\circ$ is associative. A semihypergroup I is called a hypergroup if $x \circ I = I = I \circ x$ for each $x \in I$. A non-empty subset J of a semihypergroup $(I, \circ)$ refers to a subhypergroup if $x \circ J = J = J \circ x$ for each $x \in J$.

Definition 2.1. [14] An algebraic structure $(H, +, \circ)$ refers to a commutative multiplicative hyperring if

(1) $(H, +)$ is a commutative group;

(2) $(H, \circ)$ is a semihypergroup;

(3) $x \circ (y + z) \subseteq x \circ y + x \circ z$ and $(y + z) \circ x \subseteq y \circ x + z \circ x$ for every $x, y, z \in H$;

(4) $x \circ (-y) = (-x) \circ y = -(x \circ y)$ for every $x, y \in H$;

(5) $x \circ y = y \circ x$ for every $x, y \in H$.

If in (3), the equality holds then the multiplicative hyperring H is called strongly distributive.

Conseder the ring of integers $(\mathbb{Z}, +, \cdot)$. For each subset $X \in P^\star(\mathbb{Z})$ with $|X| \geq 2$, there exists a multiplicative hyperring $(\mathbb{Z}_X, +, \circ)$ where $\mathbb{Z}_X = \mathbb{Z}$ and $a \circ b = \{a.x.b \mid x \in X\}$ for all $a, b \in \mathbb{Z}_X$ [12].

Definition 2.2. [1] An element $e \in H$ refers to a scalar identity element if $a = a \circ e$ for all $a \in H$. Moreover, an element $e \in H$ is considered as an identity element if $a \in a \circ e$ for all $a \in H$.

Throughout this paper, H denotes a commutative multiplicative hyperring with identity 1.

Definition 2.3. [14] A non-empty subset A of H is a hyperideal if

(i) $x - y \in A$ for all $x, y \in A$;

(ii) $r \circ x \subseteq A$ for all $x \in A$ and $r \in H$.

Definition 2.4. [12] A proper hyperideal A in H refers to a prime hyperideal if $x \circ y \subseteq A$ for $x, y \in H$, then $x \in A$ or $y \in A$.

The intersection of all prime hyperideals of H containing a hyperideal A is said to be the prime radical of A, denoted by $rad(A)$. If the multiplicative hyperring H has no prime hyperideal containing A, we define $rad(A) = H$. Assume that $\mathcal{C}$ is the class of all finite products of elements of H that is $\mathcal{C} = \{a_1 \circ a_2 \circ \cdots \circ a_n \mid a_i \in H, n \in \mathbb{N}\} \subseteq P^*(H)$ and A is a hyperideal of H. A refers to a $\mathcal{C}$-hyperideal of H if for each $J \in \mathcal{C}$ and $A \cap J \neq \varnothing$ imply $J \subseteq A$. Notice that $\{a \in H \mid a^n \subseteq A$ for some $n \in \mathbb{N}\} \subseteq rad(A)$. The equality holds if A is a $\mathcal{C}$-hyperideal of H (see Proposition 3.2 in [12]). Moreover, a hyperideal A of H refers to a strong $\mathcal{C}$-hyperideal if for each $E \in \mathfrak{U}$ and $E \cap A \neq \varnothing$ imply $E \subseteq A$ such that $\mathfrak{U} = \{\sum_{i=1}^{n} J_i \mid J_i \in \mathcal{C}, n \in \mathbb{N}\}$ and $\mathcal{C} = \{a_1 \circ a_2 \circ ... \circ a_n \mid a_i \in H, n \in \mathbb{N}\}$ (for more details see [13]).

Definition 2.5. [12] A proper hyperideal A in H refers to a primary hyperideal if $x \circ y \subseteq A$ for $x, y \in H$, then $x \in A$ or $y^n \subseteq A$ for some $n \in \mathbb{N}$.

Definition 2.6. [1] A proper hyperideal A of H is maximal in H if for each hyperideal B of H with $A \subset B \subseteq H$, then $B = H$.

Also, H refers to a local multiplicative hyperring if it has just one maximal hyperideal.

Definition 2.7. [1] Assume that A and B are hyperideals of H. We define $(A : B) = \{x \in H \mid x \circ B \subseteq A\}$.

3 (α,β)-Prime Hyperideals

This section discusses the fundamental characteristics of (α,β)-prime hyperideals and examines their behavior in several classes of commutative multiplicative hyperrings. We start with the following definition.

Definition 3.1. Let $\alpha,\beta \in \mathbb{Z}^+$. A proper hyperideal P of H is called (α,β)-prime if $x^\alpha \circ y \subseteq P$ for $x, y \in H$ implies $x^\beta \subseteq P$ or $y \in P$.

Example 3.2. Consider the multiplicative hyperring $\mathbb{Z}_A$. Let p is a prime integer such that $A \cap \langle p\rangle = \varnothing$. Then $\langle p\rangle$ is an (α,β)-prime for all $\alpha,\beta \in \mathbb{Z}^+$.

Remark 3.3. Let $\alpha,\beta \in \mathbb{Z}^+$.

(i) Every (α,β)-prime hyperideal of H is (α,β)-closed.

(ii) Every (α,β)-prime hyperideal of H is primary.

Proof. (i) Let P be an (α,β)-prime hyperideal of H and $x^\alpha \subseteq P$ for $x \in H$. Then we have $x^\alpha \circ 1 \subseteq P$. Since P is an (α,β)-prime hyperideal of H and $1 \notin P$, we get $x^\beta \subseteq P$, as needed.

(ii) Assume that P is an (α,β)-prime hyperideal of H and $x \circ y \subseteq P$ for $x, y \in H$. Therefore we get $x^\alpha \circ y \subseteq P$. This implies that $x^\beta \subseteq P$ or $y \in P$ as P is an (α,β)-prime hyperideal of H. Thus P is a primary hyperideal of H. □

The following example shows that the converse of statements in Remark 3.3 may not be true, in general.

Example 3.4. Consider the multiplicative hyperring $(\mathbb{Z}_X, +, \circ)$.

(i) Let $X = \{2, 3\}$. Then $P = \langle 6\rangle$ is an $(3,2)$-closed hyperideal of $\mathbb{Z}$, but it is not $(3,2)$-prime as $2^3 \circ 3 \subseteq P$ where neither $2^2 = \{8, 12\} \nsubseteq P$ nor $3 \notin P$.

(ii) Let $X = \{2, 4\}$. Then $P = \langle 8 \rangle$ is a primary hyperideal of the multiplicative hyperring $(\mathbb{Z}, +, \circ)$. However, it is not $(4, 3)$-prime as the fact that $1^4 \circ 4 \subseteq P$ but $1^3 = \{4, 8, 16\} \nsubseteq P$ and $4 \notin P$.

Recall from [4] that a proper hyperideal P of H is n-absorbing if $x_1 \circ \cdots \circ x_n \circ x_{n+1} \subseteq P$ for $x_1, \cdots, x_n, x_{n+1} \in H$, then there are n of the $x_i^,$ whose product is in P. Moreover, a proper hyperideal P of H is called semi n-absorbing if for $x \in H$, $x^{n+1} \subseteq P$ implies $x^n \subseteq P$. Let $x^{\beta+1}$ be a subset of an arbitrary (α, β)-prime hyperideal P in H for some $x, y \in H$. Then $x^\alpha \circ x = x^{\alpha-\beta} \circ x^{\beta+1} \subseteq P$. Hence we get $x^\beta \subseteq P$ or $x \in P$ as P is an (α, β)-prime hyperideal of H. Then we conclude that every (α, β)-prime hyperideal is semi β-absorbing. However, the converse need not to be hold. See the following example.

Example 3.5. In the commutative multiplicative hyperring $\mathbb{Z}_X$ with $X = \{7, 11\}$, the hyperideal $P = \langle 30 \rangle$ is 3-absorbing but it is not an $(4, 3)$-prime hyperideal of $\mathbb{Z}_X$. Because $2^4 \circ 15 \subseteq P$ but neither $2^3 \subseteq P$ nor $15 \in P$.

Remark 3.6. A product of the (α, β)-prime hyperideals may not be an (α, β)-prime hyperideal.

The following example verifies this claim.

Example 3.7. Consider the multiplicative hyperring $H = \mathbb{Z} + 3x\mathbb{Z}[x]$ defined in Example 2.4 of [16]. In the hyperring $P = 3x\mathbb{Z}[x]$ is (α, β)-prime hyperideal for all $\alpha, \beta \in \mathbb{Z}^+$, but P^n is not an (α, β)-prime hyperideal of H for $n \leq \alpha$. Because $3^\alpha x^\alpha \subseteq P^n$, $3^\beta = \{3^\beta \cdot 2^i \cdot 4^j \mid i, j \geq 0 \text{ and } i + j = \beta - 1\} \nsubseteq P^n$ and $x^\alpha \notin P^n$.

Our first theorem presents a characterization of (α, β)-prime hyperideals

Theorem 3.8. *Assume that $P \neq H$ is a hyperideal of H and $\alpha, \beta \in \mathbb{Z}^+$. Then the following are equivalent:*

(i) *P is an (α, β)-prime hyperideal in H.*

(ii) *$P = (P : x^\alpha)$ such that $x^\beta \nsubseteq P$ for $x \in H$.*

(iii) *If $x^\alpha \circ P' \subseteq P$ for some hyperideal P' of H and $x \in H$, then $x^\beta \subseteq P$ or $P' \subseteq P$.*

Proof. (i) $\Longrightarrow$ (ii) Suppose that $x^\beta \nsubseteq P$ for $x \in H$. Take any $y \in (P : x^\alpha)$. So we have $x^\alpha \circ y \subseteq P$. Since P is an (α,β)-prime hyperideal in H and $x^\beta \nsubseteq P$, we get $y \in P$ which means $(P : x^\alpha) \subseteq P$. Since the inclusion $P \subseteq (P : x^\alpha)$ always holds, we obtain $P = (P : x^\alpha)$.

(ii) $\Longrightarrow$ (iii) Let P' be a hyperideal of H and $x \in H$ such that $x^\alpha \circ P' \subseteq P$. If $x^\beta \subseteq P$, we are done. If $x^\beta \nsubseteq P$, we get the result that $(P : x^\alpha) = P$ by (ii), and so $P' \subseteq P$.

(iii) $\Longrightarrow$ (i) Let $x^\alpha \circ y \subseteq P$ for $x, y \in H$. Then $\langle x^\alpha\rangle \circ \langle y\rangle \subseteq \langle x^\alpha \circ y\rangle \subseteq P$ by Proposition 2.15 in [12]. Put $\langle y\rangle = P'$. Hence we have $x^\alpha \circ P' \subseteq P$. From (iii) it follows that either $x^\beta \subseteq P$ or $y \in P' \subseteq P$, as needed. $\square$

Recall from [12] that a hyperideal A of H refers to a principal hyperideal if $A = \langle x\rangle$ for $x \in H$. A hyperring whose every hyperideal is principal is called principal hyperideal hyperring.

Proposition 3.9. *Let $P \neq H$ be a hyperideal of a principal hyperideal hyperring H and $\alpha, \beta \in \mathbb{Z}^+$. Then the following statements are equivalent:*

(i) *P is an (α,β)-prime hyperideal in H.*

(ii) *If $P_1^\alpha \circ P_2 \subseteq P$ for hyperideals P_1, P_2 of H, then $P_1^\beta \subseteq P$ or $P_2 \subseteq P$.*

(iii) *$P = (P : P_1^\alpha)$ such that $P_1^\beta \nsubseteq P$ for every hyperideal P_1 of H.*

(iv) *If $P_1^\alpha \circ y \subseteq P$ where $y \in H$ and P_1 is a hyperideal of H, then $P_1^\beta \subseteq P$ or $y \in P$.*

Proof. (i) $\Longrightarrow$ (ii) Let $P_1^\alpha \circ P_2 \subseteq P$ for hyperideals P_1, P_2 of H. Since H is a principal hyperideal hyperring, there exists $x \in H$ such that $P_1 = \langle x\rangle$ and so $x^\alpha \circ P_2 \subseteq P$. Since P is an (α,β)-prime hyperideal in H, by Theorem 3.8 we conclude that $x^\beta \subseteq P$ which means $P_1^\beta = \langle x\rangle^\beta \subseteq \langle x^\beta\rangle \subseteq P$ or $P_2 \subseteq P$.

(ii) $\Longrightarrow$ (iii) Let $y \in (P : P_1^\alpha)$ and $P_1^\beta \nsubseteq P$ for a hyperideal P_1 of H. Then $\langle y\rangle \subseteq (P : P_1^\alpha)$ as $(P : P_1^\alpha)$ is a hyperideal of H by Theoremm

3.8 in [1], and so $P_1^\alpha \circ \langle y \rangle \subseteq P$. By the hypothesis, we get $y \in \langle y \rangle \subseteq P$ which implies $(P : P_1^\alpha) \subseteq P$. The other containment is clear.

(iii) $\Longrightarrow$ (iv) Let $P_1^\alpha \circ y \subseteq P$ and $P_1^\beta \nsubseteq P$. Then we have $y \in (P : P_1^\alpha) = P$, as required.

(iv) $\Longrightarrow$ (i) Let $x^\alpha \circ y \subseteq P$ for $x, y \in H$. We assume that $P_1 = \langle x \rangle$. Hence we have $P_1^\alpha \circ y \subseteq \langle x \rangle^\alpha \circ \langle y \rangle \subseteq \langle x^\alpha \circ y \rangle \subseteq P$. By the assumption, we get the result that $x^\beta \subseteq P_1^\beta \subseteq P$ or $y \in P$. Consequently, P is an (α, β)-prime hyperideal in H. $\square$

By Remark 3.3, every (α, β)-prime $\mathcal{C}$-hyperideal of H is a primary hyperideal and so its radical is a prime hyperideal of H by Proposition 3.6 in [12]. Let P be an (α, β)-prime $\mathcal{C}$-hyperideal of H. Then P is referred as a Q-(α, β)-prime $\mathcal{C}$-hyperideal of H where $rad(P) = Q$.

Theorem 3.10. *If P is a Q-(α, β)-prime $\mathcal{C}$-hyperideal of H for $\alpha, \beta \in \mathbb{Z}^+$, then $Q = \{x \in H \mid x^\beta \subseteq P\}$.*

Proof. The inclusion $\{x \in H \mid x^\beta \subseteq P\} \subseteq Q$ holds. Assume that $x \in Q = rad(P)$ and n is the smallest positive integer with $x^n \subseteq P$. Then we have $x^\alpha \circ x^{n-1} \subseteq P$. Take any $y \in x^{n-1}$. Since $x^\alpha \circ y \subseteq P$ and P is an (α, β)-prime hyperideal of H, we get $x^\beta \subseteq P$ or $y \in P$. Let $y \in P$. Since P is a C-hyperideal of H and $y \in x^{n-1}$, we get the result that $x^{n-1} \subseteq P$, a contradiction. Therefore we obtain $x^\beta \subseteq P$. Then $Q \subseteq \{x \in H \mid x^\beta \subseteq P\}$, so this completes the proof. $\square$

Theorem 3.11. *Let P be a $\mathcal{C}$-hyperideal of H with an i-set and $\alpha, \beta \in \mathbb{Z}^+$ such that $Q = \{x \in H \mid x^\beta \subseteq P\}$ is a maximal hyperideal of H. Then P is a Q-(α, β)-prime hyperideal.*

Proof. Assume that $x^\alpha \circ y \subseteq P$ with $x^\beta \nsubseteq P$ for $x, y \in H$. Let $x^\alpha \subseteq Q$. Since Q is a maximal hyperideal of H, we get $x \in Q$ by Proposition 2.18 in [12]. This contradict by $x^\beta \nsubseteq P$. Then $x^\alpha \nsubseteq Q$. Let $a \in x^\alpha$ such that $a \notin Q$. Then we have $\langle a, Q \rangle = H$. So, there exists $m \in Q$ such that $1 \in \langle a \rangle + m \subseteq \langle x^\alpha \rangle + m$. Therefore $1 \in (\langle x^\alpha \rangle + m)^\beta \subseteq \Sigma_{i=0}^{\beta} \binom{\beta}{i} \langle x^\alpha \rangle^{\beta - i} \circ m^i$ and so $y \in 1 \circ y \subseteq (\Sigma_{i=0} \binom{\beta}{i} \langle x^\alpha \rangle^{\beta - i} \circ m^i) \circ y \subseteq P$. Hence P is an (α, β)-prime hyperideal. By Theorem 3.10, we conclude that P is a Q-(α, β)-prime hyperideal. $\square$

As an immediate consequence of the previous theorem, we have the following result.

Corollary 3.12. *Let Q is a maximal hyperideal of H, $\alpha, \beta \in \mathbb{Z}^+$ and $n \leq \beta$. Then Q^n is a Q-(α, β)-prime hyperideal.*

Theorem 3.13. *Assume that $P_1, \cdots, P_n$ are $\mathcal{C}$-hyperideals of H such that for every $i \in \{1, \cdots, n\}$, P_i is a Q-(α_i, β_i)-prime hyperideal with $\alpha_i, \beta_i \in \mathbb{Z}^+$. Then $\bigcap_{i=1}^n P_i$ is a Q-(α, β)-prime hyperideal in H where $\alpha \leq \min\{\alpha_1, \cdots, \alpha_n\}$ and $\beta \geq \max\{\beta_1, \cdots, \beta_n\}$.*

Proof. Suppose that $x^\alpha \circ y \subseteq \bigcap_{i=1}^n P_i$ for $x, y \in H$ such that $y \notin \bigcap_{i=1}^n P_i$. This means that at least one of the $P_i^{,}$s, say P_t, does not contain y. Since P_t is a Q-(α_t, β_t)-prime hyperideal in H and $x^{\alpha_t} \circ y \subseteq P_t$, we conclude that $x^{\beta_t} \subseteq P_t$ which implies $x \in Q$. Then we get $x^{\beta_i} \subseteq P_i$ for each $i \in \{1, \cdots, n\}$ by Theorem 3.10, and so $x^\beta \subseteq \bigcap_{i=1}^n P_i$ where $\beta \geq \max\{\beta_1, \cdots, \beta_n\}$. On the other hand, we have $rad(\bigcap_{i=1}^n P_i) = \bigcap_{i=1}^n rad(P_i) = Q$ by Proposition 3.3 in [12]. Consequently, $\bigcap_{i=1}^n P_i$ is a Q-(α, β)-prime hyperideal in H. $\square$

Let $\mathfrak{C}(P) = \{(\alpha, \beta) \in \mathbb{Z}^+ \times \mathbb{Z}^+ \mid P \text{ is } (\alpha,\beta)\text{-closed } \}$ where P is a proper hyperideal of H. Then we get $\{(\alpha, \beta) \in \mathbb{Z}^+ \times \mathbb{Z}^+ \mid 1 \leq \alpha \leq \beta\} \subseteq \mathfrak{C}(P) \subseteq \mathbb{Z}^+ \times \mathbb{Z}^+$. Furthermore, $rad(P) = P$ if and only if $\mathfrak{C}(P) = \mathbb{Z}^+ \times \mathbb{Z}^+$ [5]. Now, let us define $\mathfrak{L}(P) = \{(\alpha, \beta) \in \mathbb{Z}^+ \times \mathbb{Z}^+ \mid P \text{ is } (\alpha,\beta)\text{-prime } \}$. Let $\mathfrak{L}(P) = \mathbb{Z}^+ \times \mathbb{Z}^+$. Clearly, the hyperideal P is prime if and only if $(1, 1) \in \mathfrak{L}(P)$.

Remark 3.14. Let P be a proper hyperideal of H and $\alpha, \beta \in \mathbb{Z}^+$. If $(\alpha, \beta) \in \mathfrak{L}(P)$, then $(\alpha', \beta') \in \mathfrak{L}(P)$ for $\alpha', \beta' \in \mathbb{Z}^+$ such that $\alpha' \leq \alpha$ and $\beta \leq \beta'$ by being P a hyperideal.

Remark 3.15. Suppose that P is a proper hyperideal of H and $\alpha, \beta, \theta \in \mathbb{Z}^+$. If $(\alpha, \beta) \in \mathfrak{L}(P)$ and $(\beta, \theta) \in \mathfrak{C}(P)$, then $(\alpha, \theta) \in \mathfrak{L}(P)$.

Theorem 3.16. *Assume that P is a proper hyperideal of H and $\alpha, \beta \in \mathbb{Z}^+$. Then $(\alpha, \beta) \in \mathfrak{L}(P)$ if and only if $(\alpha + 1, \beta) \in \mathfrak{L}(P)$.*

Proof. ($\Longrightarrow$) Let $(\alpha, \beta) \in \mathfrak{L}(P)$. Assume that $x^{\alpha+1} \circ y \subseteq P$ for $x, y \in H$ such that $y \notin P$. Since $\alpha + 1 \leq 2\alpha$, we have $(x^2)^\alpha \circ y \subseteq P$. Since $(\alpha, \beta) \in \mathfrak{L}(P)$ and $y \notin P$, we get $x^{2\beta} \subseteq P$ which means $x \in rad(P)$. By Theorem 3.10, we conclude that $x^\alpha \subseteq P$.

($\Longleftarrow$) It follows from Remark 3.14. $\square$

Theorem 3.17. *Let the zero hyperideal of H be a $\mathcal{C}$-hyperideal and $\alpha, \beta \in \mathbb{Z}^+$. If every proper hyperideal of H is (α, β)-prime, then H has no non-trivial idempotents, and every prime $\mathcal{C}$-hyperideal of H is maximal.*

Proof. Let every proper hyperideal of H be (α, β)-prime. Assume that e is a non-trivial idempotent in H. Then $0 \in e \circ (e-1)$ and so $0 \in e^\alpha \circ (e-1)$. Since the zero hyperideal of H is a $\mathcal{C}$-hyperideal, we have $e^\alpha \circ (e-1) = 0$. Therefore we get $e \in e^\beta = 0$ as the zero hyperideal of H is (α, β)-prime and $e \neq 1$. This is a contradiction and so H has no non-trivial idempotents. Suppose that P is a prime $\mathcal{C}$-hyperideal that is not maximal. Then we have $P \subset Q$ for some hyperideal Q of H. Let $x \in Q - P$. Then $x^\alpha \circ x \subseteq \langle x^{\alpha+1} \rangle$. This follows that $x^\beta \subseteq \langle x^{\alpha+1} \rangle$ or $x \in \langle x^{\alpha+1} \rangle$. This implies that $x^\beta \subseteq x^{\alpha+1} \circ a$ for some $a \in H$ or $x \in x^{\alpha+1} \circ b$ for some $b \in H$. In the first case, $0 \in x^\beta - x^{\alpha+1} \circ a$. Since P is a $\mathcal{C}$-hyperideal, we get $x^\beta - x^{\alpha+1} \circ a \subseteq P$. From $x^\beta \circ (1 - x^{\alpha-\beta+1} \circ r) \subseteq x^\beta - x^{\alpha+1} \circ a$ it follows that $x^\beta \circ (1 - x^{\alpha-\beta+1} \circ r) \subseteq P$. Since P is a prime hyperideal and $x^\beta \nsubseteq P$, we get $1 - x^{\alpha-\beta+1} \circ r \subseteq P \subset Q$ and so $1 \in Q$, a contradiction. In the second case, we get $0 \in (x - x^{\alpha+1} \circ b) \cap P$. Then we get the result that $x \circ (1 - x^\alpha \circ b) \subseteq x - x^{\alpha+1} \circ b \subseteq P$. Thus we have $1 - x^\alpha \circ b \subseteq P \subset Q$ because $x \notin P$. This follows that $1 \in Q$ which is a contradiction. Consequently, every prime $\mathcal{C}$-hyperideal of H is maximal. $\square$

We say that a hyperideal P of H is of maximum length β if for every ascending chain $P = P_0 \subseteq P_1 \subseteq P_2 \subseteq \cdots$ of hyperideals of H, β is the largest integer with $P_\beta = P_{\beta+1} = \cdots$.

Theorem 3.18. *Assume that $\alpha, \beta \in \mathbb{Z}^+$ and P is a strong $\mathcal{C}$-hyperideal of H of maximum length β. If P is irreducible in H, then P is an (α, β)-prime hyperideal.*

Proof. Let $x^\alpha \circ y \subseteq P$ for $x, y \in H$. Consider the ascending chain $P = P_0 \subseteq P_1 \subseteq P_2 \subseteq \cdots$ where $P_i = \{b \in H \;\; x^i \circ b \subseteq P\}$. By the assumption we have $P_\beta = P_{\beta+1} = \cdots$. Put $I = P + \langle x^\beta \rangle$ and $J = P + \langle y \rangle$. Then we have $P \subseteq I \cap J$. Let $a \in I \cap J$. Then there exist $a_1, a_2 \in H$ and $p_1, p_2 \in P$ such that $a \in (p_1 + x^\beta \circ a_1) \cap (p_2 + y \circ a_2)$. It follows that $a = p_1 + x_1 = p_2 + x_2$ for some $x_1 \in x^\beta \circ a_1$ and $x_2 \in y \circ a_2$. Therefore we get $x_1 - x_2 \in (x^\beta \circ a_1 - y \circ a_2) \cap P$ and so $x^\beta \circ a_1 - y \circ a_2 \subseteq P$ as P is a strong

$\mathcal{C}$-hyperideal of H. Since $(x^\beta \circ a_1 - y \circ a_2) \circ x^\alpha \subseteq x^{\alpha+\beta} \circ a_1 \ - x^\alpha \circ y \circ a_2$, we obtain $x^{\alpha+\beta} \circ a_1 \ - x^\alpha \circ y \circ a_2 \subseteq P$ which implies $x^{\alpha+\beta} \circ a_1 \subseteq P$. This means $a_1 \in P_{\alpha+\beta} = P_\beta$ and so $x^\beta \circ a_1 \subseteq P$. Then we conclude that $a = p_1 + x_1 \in P$ which means $I \cap J \subseteq P$ and so $P = I \cap J$. By the hypothesis, we get $P = I$ which implies $x^\beta \subseteq P$ or $P = J$ which means $y \in P$. Thus P is an (α, β)-prime hyperideal. $\square$

Recall from [17] that a proper hyperideal P of H is a 1-absorbing prime if $x \circ y \circ z \subseteq P$ for every non-unit elements $x, y, z \in H$, then $x \circ y \subseteq P$ or $z \in P$.

Theorem 3.19. *Let P be a 1-absorbing prime $\mathcal{C}$-hyperideal of H. Then P is an (α, β)-prime hyperideal for every $\beta \geq 2$.*

Proof. Assume that P is a 1-absorbing prime $\mathcal{C}$-hyperideal of H. Take any $x, y \in H$ such that $x^\alpha \circ y \subseteq P$ but $y \notin P$. If x is unit, then $y \in 1 \circ y \subseteq 1^\alpha \circ y \subseteq (x^{-1} \circ x)^\alpha \circ y = (x^{-1})^\alpha \circ x^\alpha \circ y \subseteq P$, a contradiction. Then x is nonunit. Let y be unit. Then we have $x^{\alpha-2} \circ x \circ x = x^\alpha \subseteq x^\alpha \circ 1 \subseteq x^\alpha \circ y \circ y^{-1} \subseteq P$. Take any $a \in x^{\alpha-2}$. Then we get $a \circ x \circ x \subseteq P$. If a is unit, then we are done. Let a be non-unit. Since P is a 1-absorbing prime hyperideal of H, we obtain $a \circ x \subseteq P$ or $x \in P$. In the first case, since P is a $\mathcal{C}$-hyperideal and $x^{\alpha-2} \circ x \cap P \neq \varnothing$, we have $x^{\alpha-1} = x^{\alpha-2} \circ x \subseteq P$. By continuing this process, we conclude that $x^2 \subseteq P$ which implies $x^\beta \subseteq P$ for every $\beta \geq 2$. Now, let y be non-unit and b be any element in $x^{\alpha-1}$. If b is unit, then we have $x \circ y \subseteq e \circ x \circ y \subseteq b^{-1} \circ b \circ x \circ y \subseteq b^{-1} \circ x^\alpha \circ y \subseteq P$. This implies that $x^2 \circ y \subseteq P$ which means $x^2 \subseteq P$ and so $x^\beta \subseteq P$ for every $\beta \geq 2$. Assume that b is non-unit. Since $b \circ x \circ y \subseteq P$, $y \notin P$ and P is a 1-absorbing prime hyperideals of H, we get the result that either $b \circ x \subseteq P$ which means $x^\alpha \subseteq P$. By using a similar argument mentioned above, we conclude that $x^\beta \subseteq P$ for every $\beta \geq 2$. Thus P is an (α, β)-prime hyperideal for every $\beta \geq 2$. $\square$

A proper hyperideal P of H is called semiprime, if $x^k \circ y \subseteq P$ for $k \in \mathbb{Z}$ and $x, y \in H$ implies $x \circ y \subseteq P$ [15]. The following theorem shows that the converse of Theorem 3.19 holds when P is a semiprime hyperideal.

Theorem 3.20. *Let P be a semiprime $\mathcal{C}$-hyperideal of H. If P is an (α, β)-prime for all $\beta \geq 2$, then P is 1-absorbing prime.*

Proof. Let $x \circ y \circ z \subseteq P$ for some non-unit elements $x, y, z \in H$ but $z \notin P$. Then we have $(x \circ y)^\alpha \circ z \subseteq P$. Take any $a \in x \circ y$. Therefore $a^\alpha \circ z \subseteq P$. Since P is an (α, β)-prime hyperideal of H and $z \notin P$, we get $a^\beta \subseteq P$. Since P is a $\mathcal{C}$-hyperideal and $(x \circ y)^\beta \cap P \neq \varnothing$, we obtain $x^\beta \circ y^\beta \subseteq P$. Take any $b \in y^\beta$. Then we have $x^\beta \circ b \subseteq P$. Since P is a semiprime hyperideal, we get the result that $x \circ b \subseteq P$. This means that $(x \circ y^\beta) \cap P \neq \varnothing$ and so $x \circ y^\beta \subseteq P$. This implies that $x \circ y \subseteq P$ as P is a semiprime hyperideal of H. Consequently, P is a 1-absorbing prime hyperideal of H. $\square$

Assume that $P_1, \cdots, P_n$ are hyperideals of H. A covering $A \subseteq \cup_{i=1}^n P_i$ is efficient if P is not contained in the union of any $n-1$ of the hyperideals $P_1, \cdots, P_n$. Moreover, $P = \cup_{i=1}^n P_i$ is an efficient union when none of the $P_i^,$s may be excluded [16]. The following lemma is needed in the proof of our next result.

Lemma 3.21. *Assume that $P_1, \cdots, P_n$ $(n \geq 2)$ are $\mathcal{C}$-hyperideals of H such that $P \subseteq \cup_{i=1}^n P_i$ is an efficient covering. If $rad(P_i) \nsubseteq rad((P_j : a))$ for every $a \notin rad(P_j)$ with $i \neq j$, then no P_i is an (α, β)-prime hyperideal for each $i \in \{1, \cdots, n\}$.*

Proof. Assume that one of the $P_i^,$s, say P_t, is an (α, β)-prime hyperideal of H. Clearly, $P = (P \cap P_1) \cup \cdots \cup (P \cap P_n)$ is an efficient union. Then we conclude that $(\cap_{i \neq t} P_i) \cap P \subseteq P_t \cap P$. Let $a \notin rad(P_t)$ and $i \neq t$. By the hypothesis, there exists $x_i \in rad(P_i)$ such that $x_i \notin rad((P_t : a))$. Therefore we may suppose that α_i is the least positive integer with $x_i^{\alpha_i} \subseteq P_i$. Put $X = x_1^\alpha \circ \cdots \circ x_{t-1}^\alpha \circ x_{t+1}^\alpha \circ \cdots \circ x_n^\alpha$ where $\alpha = \max\{\alpha_1, \cdots, \alpha_{t-1}, \alpha_{t+1}, \cdots, \alpha_n\}$. Thus $X \circ a \subseteq \cap_{i \neq t}(P_i \cap P)$. Now, assume that $X \circ a \subseteq P_t \cap P$. This implies that $X \subseteq (P_t : a) \subseteq rad(P_t : a)$. By Theorem 3.8, $rad(P_t : a)$ is a prime hyperideal. Then there exists some $i \in \{1, \cdots, t-1\} \cup \{t+1, \cdots, n\}$ such that $x_i \in rad(P_t : a)$, wich is impossible. Then $X \circ a \nsubseteq P \cap P_i$ which means $(\cap_{i \neq t} P_i) \cap P \nsubseteq P_t \cap P$, a contradiction. Consequently, no P_i is an (α, β)-prime hyperideal for each $i \in \{1, \cdots, n\}$. $\square$

Now, we give a generalization of Prime Avoidance Theorem for (α, β)-prime hyperideals.

Theorem 3.22. *Assume that $P_1, \cdots, P_n$ $(n \geq 2)$ are $\mathcal{C}$-hyperideals of H such that at most two of them are not (α, β)-prime. If P is a hyperideal* *of H with $P \subseteq \cup_{i=1}^{n} P_i$ and $rad(P_i) \nsubseteq rad((P_j : a))$ for all $a \notin rad(P_j)$ and $i \neq j$, then $P \subseteq P_t$ for some $t \in \{1, \cdots, n\}$.*

Proof. Suppose on the contrary that $P \nsubseteq P_t$ for all $t \in \{1, \cdots, n\}$. Let $P \subseteq \cup_{i=1}^{n} P_i$ be a covering such that at least $n-2$ of the hyperideals $P_1, \cdots, P_n$ are (α, β)-prime. Without loss of generality, one may reduce the covering to an efficient covering. If $n = 2$, then the covering is not efficient. Assume that $n > 2$. Since $rad(P_i) \nsubseteq rad((P_j : a))$ for all $a \notin rad(P_j)$ and $i \neq j$ and the covering is efficient, we conclude that no P_i is an (α, β)-prime hyperideal for each $i \in \{1, \cdots, n\}$ by Lemma 3.21. This contradicts the fact that at most two of the hyperideals $P_1, \cdots, P_n$ are not (α, β)-prime. Hence $P \subseteq P_t$ for some $t \in \{1, \cdots, n\}$. $\square$

Let S be a non-empty subset of H with scalar identity 1. Recall from [1] that S is a multiplicative closed subset (briefly, MCS), if S is closed under the hypermultiplication and S contains 1. [23] Consider the set $(H \times S/\sim)$ of equivalence classes, being denoted by $S^{-1}H$, such that $(x_1, t_1) \sim (x_2, t_2)$ if and only if there exists $t \in S$ with $t \circ t_1 \circ x_2 = t \circ t_2 \circ x_1$. The equivalence class of $(x, t) \in H \times S$ is denoted by $\frac{x}{t}$. The set $S^{-1}H$ is a multiplicative hyperring where the operation $\oplus$ and the multiplication $\odot$ are defined by

$$\frac{x_1}{t_1} \oplus \frac{x_2}{t_2} = \frac{t_1 \circ x_2 + t_2 \circ x_1}{t_1 \circ t_2} = \{\frac{a+b}{c} \mid a \in t_1 \circ x_2, b \in t_2 \circ x_1, c \in t_1 \circ t_2\}$$
$$\frac{x_1}{t_1} \odot \frac{x_2}{t_2} = \frac{x_1 \circ a_2}{t_1 \circ t_2} = \{\frac{a}{b} \mid a \in x_1 \circ x_2, b \in t_1 \circ t_2\}$$

The localization map $\pi : H \longrightarrow S^{-1}H$, defined by $x \mapsto \frac{x}{1}$, is a homomorphism of hyperrings. Also, if A is a hyperideal of H, then $S^{-1}A$ is a hyperideal of $S^{-1}H$ [23].

Theorem 3.23. *Assume that P is a $\mathcal{C}$-hyperideal of H and S a MCS such that $P \cap S = \varnothing$. If P is an (α, β)-prime hyperideal of H, then $S^{-1}P$ is an (α, β)-prime hyperideal of $S^{-1}H$.*

Proof. Let $\underbrace{\frac{x}{t_1} \odot \cdots \odot \frac{x}{t_1}}_{\alpha} \odot \frac{y}{t_2} = \frac{x^\alpha \circ y}{t_1^\alpha \circ t_2} \subseteq S^{-1}P$ for some $\frac{x}{t_1}, \frac{y}{t_2} \in S^{-1}H$. So we get $\frac{a}{t} \in \frac{x^\alpha \circ y}{t_1^\alpha \circ t_2}$ for every $a \in x^\alpha \circ y = \underbrace{x \circ \cdots \circ x}_{\alpha} \circ y$ and $t \in t_1^\alpha \circ t_2 = \underbrace{t_1 \circ \cdots \circ t_1}_{\alpha} \circ t_2$. Hence $\frac{a}{t} = \frac{a'}{t'}$ for some $a' \in P$ and $t' \in S$. Then there exists $s \in S$ such that $s \circ a \circ t' = s \circ a' \circ t$. This implies $s \circ a \circ t' \subseteq P$.

Since $a \in x^\alpha \circ y$, we conclude that $s \circ a \circ t' \subseteq s \circ x^\alpha \circ y \circ t'$. Since P is a $\mathcal{C}$-hyperideal of H, we get $s \circ x^\alpha \circ y \circ t' \subseteq P$ and then $s^\alpha \circ x^\alpha \circ y \circ t'^\alpha = (s \circ x \circ t')^\alpha \circ y \subseteq P$. Take any $z \in s \circ x \circ t'$. Since $z^\alpha \circ y \subseteq P$ and P is an (α, β)-prime hyperideal of H, we have either $z^\beta \subseteq P$ or $y \in P$. In first possibily, we get $(s \circ x \circ t')^\beta \subseteq P$ as P is a $\mathcal{C}$-hyperideal of H and $z^\beta \subseteq (s \circ x \circ t')^\beta$. Hence $\frac{x^\beta}{t_1^\beta} = \frac{s^\beta \circ x^\beta \circ t'^\beta}{s^\beta \circ t_1^\beta \circ t'^\beta} \subseteq S^{-1}P$ which means $\underbrace{\frac{x}{t_1} \odot \cdots \odot \frac{x}{t_1}}_{\beta} \subseteq S^{-1}P$ or $\frac{y}{t_2} \in S^{-1}P$. This shows that $S^{-1}P$ is an (α, β)-prime hyperideal of $S^{-1}H$. $\square$

Assume that $(H, +, \circ)$ is a multiplicative hyperring and x is an indeterminate. Then $(H[x], +, \diamond)$ is a polynomail multiplicative hyperring such that $ux^n \diamond vx^m = (u \circ v)x^{n+m}$ [6].

Theorem 3.24. *Let $\alpha, \beta \in \mathbb{Z}^+$. If P is an (α, β)-prime hyperideal of $(H, +, \circ)$, then $P[x]$ is an (α, β)-prime hyperideal of $(H[x], +, \diamond)$.*

Proof. Suppose that $u(x)^\alpha \diamond v(x) \subseteq P[x]$. Without loss of generality, we may assume that $u(x) = ax^n$ and $v(x) = bx^m$ for $a, b \in H$. Hence $a^\alpha \circ bx^{\alpha n+m} \subseteq P[x]$. This means $a^\alpha \circ b \subseteq P$. Since P is an (α, β)-prime hyperideal of $(H, +, \circ)$, we get $a^\beta \subseteq P$ or $b \in P$ which implies $u(x)^\beta = (ax^n)^\beta = a^\beta x^{\beta \cdot n} \subseteq P$ or $v(x) = bx^m \in P[x]$. Consequently, $P[x]$ is an (α, β)-prime hyperideal of $(H[x], +, \diamond)$. $\square$

In view of Theorem 3.24, we have the following result.

Corollary 3.25. *Assume that P is an (α, β)-prime hyperideal of H. Then $P[x]$ is an (α, β)-prime hyperideal of $H[x]$.*

Assume that H is a multiplicative hyperring. Then the set of all hypermatrices of H is denoted by $M_m(H)$. Let $A = (A_{ij})_{m \times m}, B = (B_{ij})_{m \times m} \in P^\star(M_m(H))$. Then $A \subseteq B$ if and only if $A_{ij} \subseteq B_{ij}$[1].

Theorem 3.26. *Suppose that P is a hyperideal of H and $\alpha, \beta \in \mathbb{Z}^+$. If $M_m(P)$ is an (α, β)-prime hyperideal of $M_m(H)$, then P is an (α, β)-prime hyperideal of H.*

Proof. Let $x^\alpha \circ y \subseteq P$ for some $x, y \in H$. Then we have

$$\begin{pmatrix} x^\alpha \circ y & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \subseteq M_m(P).$$

Since $M_m(P)$ is an (α,β)-prime hyperideal of $M_m(H)$ and

$$\begin{pmatrix} x^\alpha \circ y & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} x & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \circ \cdots \circ \begin{pmatrix} x & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}}_{\alpha} \circ \begin{pmatrix} y & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix},$$

we get the result that

$$\underbrace{\begin{pmatrix} x & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \circ \cdots \circ \begin{pmatrix} x & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}}_{\beta}$$

$$= \begin{pmatrix} x^\beta & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \subseteq M_m(P)$$

or

$$\begin{pmatrix} y & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \in M_m(P).$$

Then we conclude that $x^\beta \subseteq P$ or $y \in P$. Consequently, P is an (α,β)-prime hyperideal of H. $\square$

Recall from [14] that a mapping ψ from the multiplicative hyperring $(H_1, +_1, \circ_1)$ into the multiplicative hyperring $(H_2, +_2, \circ_2)$ refers to a hyperring good homomorphism if $\psi(a +_1 b) = \psi(a) +_2 \psi(b)$ and $\psi(a \circ_1 b) = \psi(a) \circ_2 \psi(b)$ for all $a, b \in H_1$.

Theorem 3.27. *Assume that H_1 and H_2 are two multiplicative hyperrins, $\psi : H_1 \longrightarrow H_2$ a hyperring good homomorphism and $\alpha, \beta \in \mathbb{Z}^+$.*

(i) *If P_2 is an (α,β)-prime hyperideal of H_2, then $\psi^{-1}(P_2)$ is an (α,β)-prime hyperideal of H_1.*

(ii) *If ψ is surjective and P_1 is a an (α,β)-prime $\mathcal{C}$-hyperideal of H_1 with $Ker(\psi) \subseteq P_1$, then $\psi(P_1)$ is an (α,β)-prime hyperideal of H_2.*

Proof. (i) Let $x^\alpha \circ_1 x_1 \subseteq \psi^{-1}(P_2)$ for some $x, x_1 \in H_1$. Then we have $\psi(x^\alpha \circ_1 x_1) = \psi(x)^\alpha \circ_2 \psi(x_1) \subseteq P_2$ as ψ is a hyperring good homomorphism. Since P_2 is an (α,β)-prime hyperideal of H_2, we have $\psi(x^\beta) = (\psi(x))^\beta \subseteq P_2$ which means $x^\beta \subseteq \psi^{-1}(P_2)$ or $\psi(x_1) \in P_2$ which implies $x_1 \in \psi^{-1}(P_2)$. Consequently, $\psi^{-1}(P_2)$ is an (α,β)-prime hyperideal of H_1.

(ii) Let $y^\alpha \circ_2 y_1 \subseteq \psi(P_1)$ for $y, y_1 \in H_2$. Then $\psi(x) = y$ and $\psi(x_1) = y_1$for some $x, x_1 \in H_1$ because ψ is surjective. Hence $\psi(x^\alpha \circ_1 x_1) = \psi(x)^\alpha \circ_2 \psi(x_1) \subseteq \psi(P_1)$. Now, pick any $a \in x^\alpha \circ_1 x_1$. Then $\psi(a) \in \psi(x^\alpha \circ_1 x_1) \subseteq \psi(P_1)$ and so there exists $b \in P_1$ such that $\psi(a) = \psi(b)$. Then we have $\psi(a - b) = 0$ which means $a - b \in Ker(\psi) \subseteq P_1$ and so $a \in P_1$. Therefore $x^\alpha \circ_1 x_1 \subseteq P_1$ as P_1 is a $\mathcal{C}$-hyperideal. Since P_1 is an (α,β)-prime hyperideal of H_1, we obtain $x^\beta \subseteq P_1$ or $x_1 \in P_1$. This implies that $y^\beta = \psi(x^\beta) \subseteq \psi(P_1)$ or $y_1 = \psi(x_1) \in \psi(P_1)$. Thus $\psi(P_1)$ is an (α,β)-prime hyperideal of H_2. $\square$

Now, we have the following result.

Corollary 3.28. *Let P_1 and P_2 be two hyperideals of H with $P_1 \subseteq P_2$ and $\alpha, \beta \in \mathbb{Z}^+$. Then P_2 is an (α,β)-prime hyperideal of H if and only if P_2/P_1 is an (α,β)-prime hyperideal of H/P_1.*

Proof. Consider the homomorphism $\pi : H \longrightarrow H/P_1$ defined by $\pi(x) = x + P_1$. Then the claim follows from Theorem 3.27 as π is a good epimorphism. $\square$

4 Weakly (α, β)-Prime Hyperideals

In this section, we introduce the class of the weakly (α, β)-prime hyperideals as an expansion of the (α, β)-prime hyperideals and investigate some of their properties.

Definition 4.1. Assume that P is a proper hyperideal of H and $\alpha, \beta \in \mathbb{Z}^+$. P is said to be a weakly (α, β)-prime hyperideal if $0 \notin x^\alpha \circ y \subseteq P$ for $x, y \in H$ implies that $x^\beta \subseteq P$ or $y \in P$.

Example 4.2. Consider the ring $(\mathbb{Z}_8, \oplus, \odot)$ where $\bar{x} \oplus \bar{y}$ and $\bar{x} \odot \bar{y}$ are remainder of $\frac{x+y}{8}$ and $\frac{x \cdot y}{8}$, respectively, where $+$ and $\cdot$ are ordinary addition and multiplication for all $\bar{x}, \bar{y} \in \mathbb{Z}_8$. Define the hyperoperation $\bar{x} \circ \bar{y} = \{\overline{xy}, \overline{2xy}, \overline{3xy}, \overline{4xy}, \overline{5xy}, \overline{6xy}, \overline{7xy}\}$. Then the hyperideal $Q = \{\bar{0}, \bar{4}\}$ of $(\mathbb{Z}_8, \oplus, \circ)$ is weakly $(3, 1)$-prime but it is not $(3, 1)$-prime.

Theorem 4.3. *Assume that the zero hyperideal of H is a $\mathcal{C}$-hyperideal such that its radical is prime. If P is a weakly (α, β)-prime $\mathcal{C}$-hyperideal of H for $\alpha, \beta \in \mathbb{Z}^+$, then $rad(P)$ is prime. In particular, $x^\beta \subseteq P$ for each $x \in rad(P) - rad(0)$.*

Proof. Let $x \circ y \subseteq rad(P)$ for $x, y \in H$. Then there exists $n \in \mathbb{Z}^+$ such that $x^n \circ y^n \subseteq P$. Assume that $a \in x^n$ and $b \in y^n$, so $a^\alpha \circ b \subseteq P$. If $0 \in a^\alpha \circ b$, then $a^\alpha \circ b = 0$ as the zero hyperideal of H is a $\mathcal{C}$-hyperideal. Since radical of the hyperideal is prime, we have $a \in rad(0)$ or $b \in rad(0)$. This means $x \in rad(0)$ or $y \in rad(0)$. Then we get the result that $x \in rad(P)$ or $y \in rad(P)$. Assume that $0 \notin a^\alpha \circ b$. Since P is a weakly (m, n)-prime hyperideal of H, we get either $a^\beta \subseteq P$ or $b \in P$. Since P is a $\mathcal{C}$-hyperideal, $a^\beta \subseteq x^{\beta n}$ or $b \in y^n$, we have $x^{\beta n} \subseteq P$ which implies $x \in rad(P)$ or $y^n \subseteq P$ which means $y \in rad(P)$, as needed. Now, take any $x \in rad(P) - rad(0)$. Assume that n is the least positive integer with $x^n \subseteq P$. Since $x \notin rad(0)$, we conclude that $0 \notin x^\alpha \circ x^{n-1}$. Let $y \in x^{n-1}$. So $0 \notin x^\alpha \circ y \subseteq P$. Since P is a weakly (m, n)-prime hyperideal and $y \notin P$, we get $x^\beta \subseteq P$, as required. $\square$

Theorem 4.4. *If every proper hyperideal of H is weakly (α, β)-prime such that $\alpha, \beta \in \mathbb{Z}^+$ and $\alpha \geq \beta$, then every prime $\mathcal{C}$-hyperideal of H is maximal.*

Proof. Suppose that I is a prime $\mathcal{C}$-hyperideal such that it is not maximal. Let J be a proper hyperideal such that $I \subset J$. Take any $x \in J - I$. Put $P = \langle x^{\alpha+1} \rangle$. Therefore we have $0 \notin x^\alpha \circ x \subseteq P$. By the hypothesis, we have $x^\beta \subseteq P$ or $x \in P$. In the first possibilty, we obtan $x^\beta \subseteq x^{\alpha+1} \circ r$ for some $r \in H$ which means $0 \in x^\beta - x^{\alpha+1} \circ r \cap I$. Then $x^\beta - x^{\alpha+1} \circ r \subseteq I$, also $x^\beta \circ (1 - x^{\alpha-\beta+1} \circ r) \subseteq x^\beta - x^{\alpha+1} \circ r$. Hence $x^\beta \circ (1 - x^{\alpha-\beta+1} \circ r) \subseteq I$. Since $x^\beta \nsubseteq I$ and I is a prime hyperideal, we get $1 - x^{\alpha-\beta+1} \circ r \subseteq I \subset J$ which means $1 \in J$, a contradiction. In the second possibilty, we get a contradiction by a similar argument. Consequenntly, every prime $\mathcal{C}$-hyperideal of H is maximal. $\square$

Let P be a weakly (α, β)-prime $\mathcal{C}$-hyperideal of H and $x, y \in H$. We say that (x, y) is an (α, β)-zero of P if $0 \in x^\alpha \circ y$, $x^\beta \nsubseteq P$ and $y \notin P$.

Proposition 4.5. *Let P be a weakly (α, β)-prime $\mathcal{C}$-hyperideal of H and and (x, y) be an (α, β)-zero of P where $\alpha, \beta \in \mathbb{Z}^+$. Then the following hold:*

(i) $0 \in (x + a)^\alpha \circ y$ *for all* $a \in P$.

(ii) $0 \in x^\alpha \circ (y + a)$ *for all* $a \in P$.

(iii) *If the hyperideal zero of H is a strong $\mathcal{C}$-hyperideal, then* $x^\alpha \circ a = 0$ *for all* $a \in P$.

Proof. (i) Let (x, y) be an (α, β)-zero of P and $0 \notin (x + a)^\alpha \circ y$ for some $a \in P$. Since $0 \in x^\alpha \circ y$ and P is a $\mathcal{C}$-hyperideal of H, we conclude that $x^\alpha \circ y \subseteq P$. Therefore $0 \notin (x+a)^\alpha \circ y \subseteq x^\alpha \circ y + \Sigma_{i=1}^{\alpha} \binom{\alpha}{i} x^{\alpha-i} \circ a^i \circ y \subseteq P$. Since P is a weakly (α, β)-prime hyperideal of H and $y \notin P$, we get $(x + a)^\beta \subseteq P$. On the other hand, since (x, y) is an (α, β)-zero of P and $x^\beta \nsubseteq P$, we get the result that $(x + a)^\beta \nsubseteq P$ which is a contradiction. Thus $0 \in (x + a)^\alpha \circ y$ for all $a \in P$.

(ii) Let $0 \notin x^\alpha \circ (y + a)$ for some $a \in P$. Hence $0 \notin x^\alpha \circ (y + a) \subseteq x^\alpha \circ y + x^\alpha \circ a \subseteq P$ as P is a $\mathcal{C}$-hyperideal of H. Since P is a weakly (α, β)-prime hyperideal of H and $x^\beta \nsubseteq P$, we obtain $y + a \in P$ and so

$y \in P$ which is a contradiction. Consequently, $0 \in x^\alpha \circ (y + a)$ for all $a \in P$.

(iii) Assume that $x^\alpha \circ a \neq 0$ for some $a \in P$. Then there exists $u \in x^\alpha \circ a$ such that $u \neq 0$. By (ii) we have $0 \in x^\alpha \circ (y + a)$. Since the hyperideal zero of H is a strong $\mathcal{C}$-hyperideal and $0 \in x^\alpha \circ (y + a) \subseteq x^\alpha \circ y + x^\alpha \circ a$, we get $x^\alpha \circ y + x^\alpha \circ a = 0$. Moreover, since $0 \in x^\alpha \circ y$ and $0 \neq u \in x^\alpha \circ a$, we get $u = 0 + u \in x^\alpha \circ y + x^\alpha \circ a$ which is a contradiction. Hence $x^\alpha \circ a = 0$ for all $a \in P$. $\square$

Recall from [1] that an element $x \in G$ is nilpotent if there exists an integer t such that $0 \in x^t$. The set of all nilpotent elements of G is denoted by Υ.

Theorem 4.6. *Let P be a weakly (α, β)-prime $\mathcal{C}$-hyperideal of a strongly distributive multiplicative hyperring H and (x, y) be an (α, β)-zero of P where $\alpha, \beta \in \mathbb{Z}^+$. Then*

(i) *If the hyperideal zero of H is a strong $\mathcal{C}$-hyperideal, then $x \circ a \subseteq \Upsilon$ for all $a \in P$.*

(ii) *If the hyperideal zero of H is a $\mathcal{C}$-hyperideal, $y \circ a \subseteq \Upsilon$ for all $a \in P$.*

Proof. (i) Since $x^\alpha \circ a = 0$ for all $a \in P$, by Proposition 4.5 (3), we get the result that $x \circ a \subseteq \Upsilon$.

(ii) Take any $a \in P$. Since $0 \in (x + a)^\alpha \circ y$ by Proposition 4.5(1) and the hyperideal zero of H is a $\mathcal{C}$-hyperideal, we get $((x + a) \circ y)^\alpha = 0$. This means that $(x + a) \circ y \subseteq \Upsilon$. Also, since $0 \in x^\alpha \circ y$ and the hyperideal zero of H is a $\mathcal{C}$-hyperideal, we have $(x \circ y)^\alpha = 0$ which implies $x \circ y \subseteq \Upsilon$. Since H is a strongly distributive multiplicative hyperring, we have $a \circ y = (x + a) \circ y - x \circ y \subseteq \Upsilon$, as needed. $\square$

Theorem 4.7. *Let $\{P_i\}_{i \in I}$ be a family of weakly (α, β)-prime hyperideals of H and $D(P_i) = \{x \in H \mid x^\beta \subseteq P_i\}$ for all $i \in I$ where $\alpha, \beta \in \mathbb{Z}^+$. If $D(P_i) = D(P_j)$ for all $i, j \in I$, then $\bigcap_{i \in I} P_i$ is a weakly (α, β)-prime hyperideal of H.*

Proof. Assume that $0 \notin x^\alpha \circ y \subseteq \bigcap_{i \in I} P_i$ for $x, y \in H$ but $y \notin \bigcap_{i \in I} P_i$. Therefore we conclude that $y \notin P_j$ for some $j \in I$. Since P_j is a weakly (α, β)-prime hyperideal of H and $0 \notin x^\alpha \circ y \subseteq P_j$, we get the result that

$x^\beta \subseteq P_j$. This implies that $x \in D(P_j)$ and so $x \in D(P_i)$ for all $i \in I$ by the hypothesis. Then $x^\beta \subseteq \bigcap_{i\in I} P_i$. This shows that $\bigcap_{i\in I} P_i$ is a weakly (α,β)-prime hyperideal of H. $\square$

Let I be a finite sum of finite products of elements of H. Consider the relation γ on a multiplicative hyperring H defined as $x\gamma y$ if and only if $\{x,y\} \subseteq I$, namely, $x\gamma y$ if and only if $\{x,y\} \subseteq \sum_{j\in J}\prod_{i\in I_j} z_i$ for some $z_1,...,z_n \in H$ and $I_j, J \subseteq \{1,...,n\}$. γ^* denotes the transitive closure of γ. The relation γ^* is the smallest equivalence relation on H such that the set of all equivalence classes, i.e., the quotient G/γ^*, is a fundamental ring. Assume that Σ is the set of all finite sums of products of elements of H. We can rewrite the definition of γ^* on H, namely, $x\gamma^* y$ if and only if there exist $z_1,...,z_n \in H$ such that $z_1 = x, z_{n+1} = y$ and $u_1,...,u_n \in \Sigma$ where $\{z_i, z_{i+1}\} \subseteq u_i$ for $1 \leq i \leq n$. Suppose that $\gamma^*(x)$ is the equivalence class containing $x \in H$. Define $\gamma^*(x) \oplus \gamma^*(y) = \gamma^*(z)$ for every $z \in \gamma^*(x) + \gamma^*(y)$ and $\gamma^*(x) \odot \gamma^*(y) = \gamma^*(w)$ for every $w \in \gamma^*(x) \circ \gamma^*(y)$. Then $(H/\gamma^*, \oplus, \odot)$ is a ring called a fundamental ring of H [28].

Theorem 4.8. *Assume that P is a hyperideal of H. Then P is a weakly (α,β)-prime hyperideal of $(H,+,\circ)$ if and only if P/γ^* is a weakly (α,β)-prime ideal of $(H/\gamma^*, \oplus, \odot)$.*

Proof. $(\Longrightarrow)$ Let $0 \neq \underbrace{x \odot \cdots \odot x}_{\alpha} \odot y \in P/\gamma^*$ for some $x, y \in H/\gamma^*$. Hence we have $x = \gamma^*(a)$ and $y = \gamma^*(b)$ for some $a, b \in H$. This means that $\underbrace{x \odot \cdots \odot x}_{\alpha} \odot y = \underbrace{\gamma^*(a) \odot \cdots \odot \gamma^*(a)}_{\alpha} \odot \gamma^*(b) = \gamma^*(a^\alpha \circ b)$. Since $\gamma^*(0) \neq \gamma^*(a^\alpha \circ b) \in P/\gamma^*$, we get $0 \notin a^\alpha \circ b \subseteq P$. Since P is a weakly (α,β)-prime hyperideal of H, we get the result that $a^\beta \subseteq P$ or $b \in P$. This implies that $\underbrace{x \odot \cdots \odot x}_{\beta} = \underbrace{\gamma^*(a) \odot \cdots \odot \gamma^*(a)}_{\beta} = \gamma^*(a^\beta) \in P/\gamma^*$ or $y = \gamma^*(a) \in P/\gamma^*$. Consequently, P/γ^* is a weakly (α,β)-prime ideal of H/γ^*.

$(\Longleftarrow)$ Suppose that $0 \notin a^\alpha \circ b \subseteq P$ for some $a, b \in H$. Then we have $\gamma^*(a), \gamma^*(b) \in H/\gamma^*$ and so $\gamma^*(0) \neq \underbrace{\gamma^*(a) \odot \cdots \odot \gamma^*(a)}_{\alpha} \odot \gamma^*(b) = \gamma^*(a^\alpha \circ b) \in P/\gamma^*$. Since P/γ^* is a weakly (α,β)-prime ideal of H/γ^*, we obtain $\underbrace{\gamma^*(a) \odot \cdots \odot \gamma^*(a)}_{\beta} = \gamma^*(a^\beta) \in P/\gamma^*$ which implies $a^\beta \subseteq P$ or $\gamma^*(b) \in P/\gamma^*$ which means $b \in P$. Thus P is a weakly (α, β)-prime hyperideal of H. $\square$

Let $(H_1, +_1, \circ_1)$ and $(H_2, +_2, \circ_2)$ be two multiplicative hyperrings with nonzero identity. The set $H_1 \times H_2$ with the operation $+$ and the hyperoperation $\circ$ defined as

$(x_1, x_2) + (y_1, y_2) = (x_1 +_1 y_1, x_2 +_2 y_2)$

$(x_1, x_2) \circ (y_1, y_2) = \{(x, y) \in H_1 \times H_2 \mid x \in x_1 \circ_1 y_1, y \in x_2 \circ_2 y_2\}$

is a multiplicative hyperring [27]. Now, we present some characterizations of weakly (α, β)-prime hyperideals on cartesian product of commutative multiplicative hyperring.

Proposition 4.9. *Let $(H_1, +_1, \circ_1)$ and $(H_2, +_2, \circ_2)$ be two multiplicative hyperrings with scalar identities 1_{H_1} and 1_{H_2}, respectively, P a proper nonzero hyperideal of $H_1 \times H_2$, and $\alpha, \beta \in \mathbb{Z}^+$. If P is weakly (α, β)-prime, then it has one of the following cases:*

(i) *$P = P_1 \times H_2$ such that P_1 is an (α, β)-prime hyperideal of H_1.*

(ii) *$P = H_1 \times P_2$ such that P_2 is an (α, β)-prime hyperideal of H_2.*

Proof. Suppose that $P = P_1 \times P_2$ is a nonzero weakly (α, β)-prime hyperideal of $H_1 \times H_2$ such that P_1 and P_2 are hyperideals of H_1 and H_2, respectively. Let us assume P_1 and P_2 are proper, and $P_1 \neq 0$. Take any $0 \neq x \in P_1$. Therefore we have $(0, 0) \notin (1_{H_1}, 0)^\alpha \circ (x, 1_{H_2}) \subseteq P_1 \times P_2$. Since P is a nonzero weakly (α, β)-prime hyperideal of $H_1 \times H_2$, we get the result that $(1_{H_1}, 0)^\beta \subseteq P_1 \times P_2$ or $(x, 1_{H_2}) \in P_1 \times P_2$. It follows that $1_{H_1} \in P_1$ or $1_{H_2} \in P_2$. Then $P_1 = H_1$ or $P_2 = H_2$. This is a contradiction. Let us consider P_1 is proper and $P_2 = H_2$. Now, we shows that P_1 is an (α, β)-prime hyperideal of H_1. Let $x^\alpha \circ_1 y \subseteq P_1$ for $x, y \in H_1$. Hence we get $(0, 0) \notin (x, 1_{H_2})^\alpha \circ (y, 1_{H_2}) \subseteq P_1 \times H_2$ and so we have $(x, 1_{H_2})^\beta \subseteq P_1 \times H_2$ or $(y, 1_{H_2}) \in P_1 \times H_2$. This implies that $x^\beta \subseteq P_1$ or $y \in P_1$. Similarly, it can be seen that if $P_1 = H_1$ and P_2 is a proper hyperideal of H_2, then P_2 is (α, β)-prime. $\square$

Theorem 4.10. *Assume that $H = H_1 \times \cdots \times H_n$ where $H_1, \cdots, H_n$ are commutative multiplicative hyperrings, P a proper nonzero hyperideal of H and $\alpha, \beta \in \mathbb{Z}^+$. Then the following are equivalent.*

(i) *P is a weakly (α,β)-prime hyperideal of H.*

(ii) *$P = H_1 \times \cdots \times P_i \times \cdots \times H_n$ such that P_i is an (α,β)-prime hyperideal of H_i for some $i \in \{1,\cdots,n\}$.*

(iii) *P is an (α,β)-prime hyperideal of H.*

Proof. (i) $\Longrightarrow$ (ii) Let $P = P_1 \times \cdots \times P_n$ is a weakly (α,β)-prime hyperideal of H. We use the induction on n. If $n = 2$, then the claim is true by Proposition 4.9. Let the claim be true for $n-1$. Assume that $I = P_1 \times \cdots \times P_{n-1}$. So $P = I \times P_n$. Then we conclude that I is an (α,β)-prime hyperideal of $H_1 \times \cdots \times H_{n-1}$ and $P_n = H_n$ or $I = H_1 \times \cdots \times H_{n-1}$ and P_n is an (α,β)-prime hyperideal of H_n by Proposition 4.9. In the first possibility, we obtain $I = H_1 \times \cdots \times P_i \times \cdots \times H_{n-1}$ such that P_i is an (α,β)-prime hyperideal of H_i by induction hypothesis and $P_n = H_n$. This shows that $P = H_1 \times \cdots \times P_i \times \cdots \times H_{n-1} \times H_n$ such that P_i is an (α,β)-prime hyperideal of H_i. In the second possibility, we have $I_i = H_i$ for all $i \in \{1,\cdots,n-1\}$ and P_n is an (α,β)-prime hyperideal of H_n. It follows that $P = H_1 \times \cdots \times H_{n-1} \times P_n$ where P_n is is an (α,β)-prime hyperideal of H_n.

(ii) $\Longrightarrow$ (iii) Without loss of generality, we assume that P_1 is an (α,β)-prime hyperideal of H_1 and $P_i = H_i$ for all $i \neq 1$. Let us assume $(x_1,x_2,\cdots,x_n)^\alpha \circ (y_1,y_2,\cdots,y_n) \subseteq P_1 \times H_2 \times \cdots \times H_n$ such that $(y_1,y_2,\cdots,y_n) \notin P_1 \times H_2 \times \cdots \times H_n$. This implies that $x_1^\alpha \circ_1 y_1 \subseteq P_1$ and $y_1 \notin P_1$. Since P_1 is an (α,β)-prime hyperideal of H_1, we get $x_1^\beta \subseteq P_1$. It follows that $(x_1,x_2,\cdots,x_n)^\beta \subseteq P_1 \times H_2 \times \cdots \times H_n$ and this completes the proof.

(iii) $\Longrightarrow$ (i) It is straightforward. $\square$

5 Conclusions

In this paper, we generalized the concept of (α,β)-prime ideals in multiplicative hyperrings by introducing (α,β)-prime hyperideals. We provided several key results explaining the structure of this concept. The stability of these hyperideals in various hyperring-theoretic constructions was examined. Furthermore, we extended this notion to weakly (α,β)-prime hyperideals and presented several properties of this concept. Finally, we offered characterizations of the weakly (α,β)-prime hyperideals on the cartesian product of commutative multiplicative hyperrings.

6 Future work

Definition 6.1. Assume that $\phi : \mathcal{HI}(H) \longrightarrow \mathcal{HI}(H) \cup \{\varnothing\}$ is a map where $\mathcal{HI}(H)$ is the set of hyperideals of a commutative multiplicative hyperring H and $\alpha, \beta \in \mathbb{Z}^+$. A proper hyperideal P in H refers to a ϕ-(α, β)-prime hyperideal if $x^\alpha \circ y \subseteq P - \phi(P)$ for $x, y \in H$, then $x^\beta \subseteq P$ or $y \in P$.

References

[1] R. Ameri, A. Kordi and S. Hoskova-Mayerova, Multiplicative hyperring of fractions and coprime hyperideals, *An. St. Univ. ovidious Constanta,* 25 (1) (2017), 5-23.

[2] R. Ameri and A. Kordi, On regular multiplicative hyperrings, *Eur. J. Pure Appl. Math.,* 9 (4) (2016), 402-418.

[3] R. Ameri and A. Kordi, Clean multiplicative hyperrings, *Ital. J. Pure Appl. Math.*, (35) (2015), 625-636.

[4] M. Anbarloei, A generalization of prime hyperideals, *J. Algebr. Syst.,* 8 (1) (2020), 113-127.

[5] M. Anbarloei, (weakly) (s, n)-closed hyperideals in commutative multiplicative hyperrings, *J. Algebr. Syst.*, (2024), https://doi.org/10.22044/JAS.2024.13889.1780.

[6] R.P. Ciampi and R. Rota, Polynomials over multiplicative hyperrings, *J. Discrete Math. Sci. Cryptogr.*, 6 (2-3) (2003), 217–225.

[7] P. Corsini, *Prolegomena of hypergroup theory*, Second ed., Aviani Editore, (1993)

[8] P. Corsini and V. Leoreanu, *Applications of hyperstructures theory*, Adv. Math., Kluwer Academic Publishers, (2003)

[9] J. Chvalina, J., S. Krehlik and M. Novak, Cartesian composition and the problem of generalizing the MAC condition to quasi- multiautomata, *An. Stiint. Univ. Ovidius Constanta Ser. Mat.,* 24 (3) (2016), 79-100.

[10] I. Cristea and S. Jancic-Rasovic, Compositions hyperrings, *An. Stiint. Univ. Ovidius Constanta Ser. Mat.,* 21 (2) (2013), 81-94.

[11] I. Cristea, Regularity of intuitionistic fuzzy relations on hypergroupoids, *An. Stiint. Univ. Ovidius Constanta Ser. Mat.,* 22 (1) (2014), 105-119.

[12] U. Dasgupta, On prime and primary hyperideals of a multiplicative hyperring, *An. Stiint. Univ. Al. I. Cuza Iasi. Mat.,* LVIII (1) (2012), 19-36.

[13] U. Dasgupta, *On certain classes of hypersemirings*, PhD Thsis, University of Calcutta, (2012)

[14] B. Davvaz and V. Leoreanu-Fotea, *Hyperring theory and applications*, International Academic Press, USA, (2007)

[15] F. Farzalipour and P. Ghiasvand, Semiprime hyperideals in multiplicative hyperrings, *51th Annual Iranian Mathematics conference,* Iran, 1 (2021), 801-804.

[16] P. Ghiasvand and F. Farzalipour, On S-prime hyperideals in multiplicative hyperrings, *J. Algebr. Hyperstruct. Log. Algebras,* 2 (2) (2021), 25-34.

[17] P. Ghiasvand and F. Farzalipour, 1-absorbing prime avoidance theorem in a multiplicative hyperring, *Palest. J. Math.,* 12 (1) (2023), 900-908.

[18] L. Kamali Ardekani and B. Davvaz, Differential multiplicative hyperring, *J. Algebr. Syst.*, 2 (1) (2014), 21-35.

[19] H.A. Khashan and E.Y. Celikel, (m,n)-prime ideals of commutative rings, Preprints 2024, 2024010472. https://doi.org/10.20944/preprints202401.0472.v1.

[20] H.A. Khashan and E.Y. Celikel, On weakly (m, n)-prime ideals of commutative rings, *Bull. Korean Math. Soc.,* In press.

[21] F. Marty, Sur une generalization de la notion de groupe, *8iem Congres des Mathematiciens Scandinaves,* Stockholm, (1934), 45-49.

[22] C. G. Massouros, On the theory of hyperrings and hyperfields, *Algebra Logika,* 24 (1985), 728-742.

[23] A.A. Mena and I. Akray, n-absorbing I-prime hyperideals in multiplicative hyperrings, *J. Algebr. Syst.*, 12 (1) (2024), 105-121.

[24] M. Novak, n-ary hyperstructures constructed from binary quasi-orderer semigroups, *An. Stiint. Univ. Ovidius Constanta Ser. Mat.,* 22 (3) (2014), 147-168.

[25] R. Procesi, R. Rota, On some classes of hyperstructures, *Discrete Math.,* 208/209 (1999), 485-497.

[26] R. Rota, Sugli iperanelli moltiplicativi, *Rend. Di Math.,* VII (4) (1982), 711-724.

[27] G. Ulucak, On expansions of prime and 2-absorbing hyperideals in multiplicative hyperrings, *Turkish J. Math.,* 43 (2019), 1504-1517.

[28] T. Vougiouklis, *Hyperstructures and their representations*, Hadronic Press, Inc., Palm Harber, USA, (1994)

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