**Innovative Approaches to Centralized Resource Allocation with Customized Returns to Scale**

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**Abstract**

In the centralized data envelopment analysis (DEA) model, one unit supervises others, resulting in the concentration of all resources within the organization. Then, resource allocation to all units is performed based on the specific conditions of the organizations. If restrictions are applied on each corresponding indicator for each unit, they are also taken into account. The aim of this paper is to develop resource allocation technology, by introducing the concept of pseudo-returns to scale. So far, in the technologies presented, the development coefficient has been considered the same across the indicators. However, this is not always practical. Therefore, in this article, technologies have been proposed where the indices are categorized into different groups based on the flexibility of their development coefficients. The change coefficient for the sum of certain indicators is assumed as a multiple of the development coefficient for the sum of other indicators. Corresponding models are designed based on the proposed technologies. By interacting with system managers and solving models, resource allocation is accomplished ideally, aligning with the desired target setting. The primary focus of this paper lies in the developed of centralized data envelopment analysis with development of principle returns to scale. The designed models are implemented on a numerical example. Finally, it compares the total resources consumed by each group of input indicators and the total output produced by each group of output indicators from the proposed models with previously presented models. It also examines the percentage of resource savings and the percentage of profit obtained from solving the proposed models.

**Keywords** Centralized Data Envelopment Analysis, Pseudo-Returns to Scale Principle, Development Coefficient, Target Setting, Resource Savings.

**1 IntroductionTop of Form**

DEA is a non-parametric method used in operations research and economics to assess the efficiency of various entities, referred to as decision-making units (DMUs). Unlike parametric methods, which assume a specific functional form for the relationship between inputs and outputs, DEA does not require any assumptions about the functional form. This flexibility makes DEA a powerful tool for evaluating performance in diverse contexts. Based on this, the performance of decision-making units (DMUs) is evaluated. Additionally, DEA identifies efficient and inefficient DMUs. DEA is built upon foundational principles, and production technologies are designed based on these principles, leading to the creation of various models. Subsequently, the system's performance is assessed by solving these models. Occasionally, multiplier models are used to evaluate the efficiency of decision-making units. Various models have been proposed for evaluating DMUs with different objectives using DEA. Among the presented models, models have been suggested about on centralized resource allocation. Centralized resource allocation is an age-old research issue discussed in the field of management. Many researchers have explored this field extensively. In the literature review section, we referred to them. Significant distinction between centralized resource allocation models and other Data Envelopment Analysis (DEA) models. In centralized DEA examines the total consumption of inputs and the total production of outputs. In models related to centralized resource allocation (CRA), there exists a centralized decision-making unit who supervises all activities. Consequently, it takes into account all inputs and outputs of DMUs and allocates new inputs to these units based on the system’s requirements and size. With the aim of improving or at least not worsening the system's efficiency. It is important to note that in resource allocation for each activity, the objective is to have a total consumption of inputs that is less than or equal to before the allocation, while having a total production of outputs that is equal to or greater than before. In other words, the goal of these models is to set appropriate objectives, save on total resources, and, if possible, increase the total production in the systems. These models are solved using a single linear programming problem. The illustration of all DMUs lies on the efficiency frontier defined by themselves. Charnes and et al. (1978) introduced a linear programming method called Data Envelopment Analysis, which measures the efficiency of DMUs using estimation of production functions. In this research, the development of measuring the efficiency of decision-making units with benchmarking has been investigated. It is commonly used in the evaluation of public programs. They proposed the CCR model. Then, they defined efficiency based on the CCR model. Banker and et al. (1984) evaluate the performance of DMUs with multiple inputs and outputs by introducing and applying Variable Returns to Scale (VRS). From both technical and economic perspectives, they examine and develop the concept of variable returns to scale (increasing, decreasing, or constant). Lozano and Villa (2004) presented the concept of centralized resource allocation using DEA. With conventional centralized DEA models, set targets are achieved for each decision-making unit. Two fundamental characteristics are examined in the proposed model. The first characteristic is the location of all DMUs relatives to the efficient frontier. The second characteristic demonstrates the optimal shaping of the overall consumed input and produced output by finding weights that maximize the relative efficiency of units, equivalent to taking the averages of inputs and outputs, using a multiplier model. In the literature review section, studies conducted in the field of centralized resource allocation based on Data Envelopment Analysis have been discussed. Thus far, models have been proposed in the field of centralized resource allocation under assumptions of constant returns to scale, variable returns to scale, trade-offs, two-stage network structures, and various other frameworks. (For example: Lozano and villa, 2004; Asmild, 2009; Hosseinzadeh Lotfi and et al., 2010; Davutyan and et al., 2010; Hosseinzadeh Lotfi and et al., 2012; Fang, 2013; Feng, 2014; Yu and Chen, 2016; Yang and et al., 2018; Tao and et al., 2020; and etc.) In the direction of system development, the ratio of changes in the sum of all input indicators to the sum of output indicators has been equal or less than one or greater than one.

 When dealing with systems in which the coefficient of changes in the total of all input and output indicators is not uniform, meaning that some indicators change more or less than others, and in some cases, no change is required. So, how can we define the production technology in a centralized allocation scenario? If we intend to design a centralized resource allocation model based on data envelopment analysis for such systems, how do the results change? Can the central decision-making optimally allocate inputs and produce outputs, or set appropriate target for each unit in such a way that the sum of the allocated inputs is equal to or less than the initial total inputs, and the sum of the produced outputs is equal to or greater than the initial total outputs? Does the developed centralized resource allocation model result in more resource savings compared to other studies? Therefore, the proposed model must correspond to production technologies. Production possibility sets are created as units expand or contract during the system's development. In this research, the change coefficient for some indicators is considered as a multiple of the development coefficient of other indicators. Therefore, it is important to interact with the manager when categorizing the indicators and selecting their change coefficients. Additionally, it must be feasible from a management perspective. So, constraints have been added to the model, setting it apart from previous studies. The primary contribution of this study is as follows:

* Developing centralized resource allocation production technology involves categorizing indicators and applying distinct coefficients of variation to each group of indicators.
* Applying the principle of pseudo-returns to scale in the centralized data envelopment analysis (DEA) model.
* Examining the allocated resources and their savings based on the newly developed principles corresponding to each index and comparing them with the results of the centralized resource allocation model by Lozano and Villa (2004).
* Analyzing the strengths of the proposed centralized data envelopment analysis model in comparison to the Lozano and Villa (2004) centralized data envelopment analysis model.

 The subsequent sections of the paper are organized as follows: A literature review is presented in the second section. The third section describes the centralized data envelopment analysis model based on classical principles. The fourth section introduces the proposed centralized data envelopment analysis models with the application of the pseudo-returns to scale principle. The fifth section demonstrates the proposed models with a numerical example. The sixth section presents the obtained results from the proposed model and suggestions for future research.

**2 Literature Review**

In this section, reference is cited to the studies that have been conducted in the field of centralized resource allocation. Centralized resource allocation (CRA) has been applied in areas related to income efficiency, network structures, target setting, as well as centralized resource allocation models based on data envelopment analysis (DEA) principles, including constant returns to scale, variable returns to scale, etc. It has also been investigated on deterministic, random, and fuzzy data. Lozano and Villa (2005) introduced an approach to efficiently allocate resources in a centralized decision-making unit that is close to the operational point. They discussed three specific models due to the simplicity of input nature under BCC. The first model maximizes the reduction in the total inputs while maintaining the overall output level. It also allows the existing DMUs to get closer to efficiency. The second model maximizes the reduction in the total inputs, maintains the overall output level, and adjusts the maximum remaining operational unit. The third model reduces some of the open operational points, ensuring the minimum reduction in the total on each input and maintaining the overall output level. Asmild and et al. (2009) have proposed a modified centralized resource allocation model under the oriented input of BCC on the model presented by Lozano and Villa (2004). The presented model only focuses on the allocation of inefficient units. The presented method is designed to generate optimal solutions. The proposed model is developed for non-discretionary and non-controllable input variables. The development of this model has significant value in the field of data envelopment analysis theory, especially when analyzing decision-making processes is required. It is often applicable to organizations with centralized control over operational factors. Lozano et al. (2009) introduced a DEA method for reallocating emission permits, which can be used in both traditional regulation and market-based systems. The model assumes that firms produce both desirable (good) and undesirable (bad) outputs. The method aims to achieve three main goals: increasing desirable outputs, reducing undesirable emissions, and minimizing resource use. These objectives are prioritized by the regulator. Importantly, the approach is independent of input and output prices and operates effectively regardless of the measurement units used. Lozano and et. al. (2011) proposed a data envelopment analysis approach for target setting and resource allocation for the National Ports Organization in Spain. They introduced a non-radial centralized data envelopment analysis model in the oriented-output. Fang (2013) offered a generalized CRA-BCC model based on the Lozano and Villa (2004) and Asmild et al. (2009) models. Yu and et al. (2013) developed a centralized resource allocation model using a two-phase process. They also demonstrated that each DMU and the central unit are projected onto the efficiency frontier. In the model proposed by Yu and et al. (2013), due to the two-phase process, they face challenges due to model incompatibilities. Therefore, the modified model by Yu and Hasyim is presented. Mirsalehy and et al. (2014) introduced an alternative approach to centralized resource allocation for each DMU. The proposed approach offered a method for integrating the two fundamental models, Radial CRA-BCC and Non-radial CRA-SBM, into a unified framework termed connected CRA-SBM. In this model, adjusting the parameters enables shifting the analysis between CRA-BCC and CRA-SBM models, addressing inherent weaknesses in both models. They called it the integrated CRA-SBM structure. In the proposed model, both inputs and outputs are simultaneously reduced and increased. With this proposed model, the image of all decision-making units is placed on the efficient frontier. Shamsi and et al. (2014) have proposed centralized resource allocation using a multi-objective linear programming framework. They solve the multi-objective programming model using the entropy and Zionts-Wallenius (Z-W) method. This research aims to provide insights into non-cooperative DMUs. By considering environmental constraints that impact economic growth, the study investigates the problem. The allocation of carbon emission reduction is a significant concern in this context. To date, data envelopment analysis techniques have been used to address carbon emission reduction allocation through centralized resource allocation. Therefore, Feng and et al. (2014) have addressed the reduction of carbon emission allocation (CEA) and its side effects in a two-stage process. In the first stage, they have introduced an improved centralized resource allocation model based on data envelopment analysis under the assumptions of constant returns to scale and variable returns to scale. In the second stage, they have proposed two compensation schemes for centralized resource allocation programs. Yu and Hasio (2015) have presented a slacks-based centralized DEA model for resource allocation in a single phase. They modified a single-phase slack-based CDEA model to incorporate transfer-in and transfer-out slacks, thus enabling the reallocation and adjustment of resources more effectively. Sun and et al. (2015) have used a centralized DEA approach to find an optimal path for controlling the emissions of a portion of $CO\_{2}$ among regions in China. The proposed model examines environmental production technology and focuses on desirable centralized outputs while allocating the undesirable outputs. They developed this model based on the enhanced Kuosmanen environmental DEA technology, which circumvents the issue of positive shadow prices on undesirable outputs. Additionally, they devised a dual model for our centralized DEA model and utilized it to examine shadow prices associated with CO2 emissions. Subsequently, they applied the proposed model to ascertain the optimal strategy for controlling CO2 emissions at the sector level across each province in China. At the sectoral level, their analysis revealed that the manufacturing sector exhibited the greatest potential for emissions reduction, while the transportation sector emerged as the primary recipient of emission quotas. An and et al. (2016) proposed two-stage network-structured models considering centralized degree. This approach allowed for measuring the internal wastage of resources within systems and increasing the outputs. The proposed method is suitable for complex network-structured systems. Lopez-Torres and Prior (2016) introduced an alternative model for the reallocation of human resources in a public education network, which they called centralized resource allocation. They made changes to maintain the additional education budget without compromising the outputs, with the aim of improving school performance while ensuring that the quality of education remains intact. Fang (2016) expanded upon the centralized DEA models proposed by Lozano et al. (2011) to allocate resources according to revenue efficiency across a set of DMUs within a centralized decision-making framework. His objective was to allocate resources in a manner that maximizes the total output revenue generated by all DMUs while operating under limited information. In order to elucidate the factors contributing to the increase in total revenue resulting from the centralized resource allocation model, He further broke down aggregate revenue efficiency into three parts: aggregate output-oriented technical efficiency, aggregate output allocative efficiency, and aggregate revenue re-allocative efficiency. Hakim and et al. (2016) proposed a two-level data envelopment analysis model for centralized resource allocation, considering upper and lower bounds for decision-making units. The advantage of this model lies in its consideration of efficiency and effectiveness for resource allocation. In the upper-level model, it allocates input resources along the path that maximizes the effectiveness of organizations, while ensuring that the lower bound applies to the efficiency of all decision-making units. In the lower-level models, data envelopment analysis is used to determine the efficiency of decision-making units under BCC separately. Mottaghi and et al. (2017) applied resource allocation to systems with optional and non-discretionary inputs, taking into account environmental factors. They proposed a multi-objective linear programming model for resource allocation. Zhou and et al. (2017) presented a production possibility set for a two-stage network structure with random data. They suggested data envelopment analysis to the two-stage network structure with random data under centralized control management. The presented model is a deterministic linear programming model. Ding and et. al. (2018) introduced a new approach under a centralized decision-making environment to address fixed costs and resource allocation problems while considering the factor of technological heterogeneity. The proposed models are based on constant returns to scale (CCR) assumption. They introduced the concepts of non-discretionary, border group, and meta-technology ratio. The level of technology reflects the decision-making units. Two centralized data envelopment analysis models under technological heterogeneity have been suggested. Yang and et al. (2018) proposed CRA and target setting based on data envelopment analysis. In this paper, they used the CCR model and extended the approach to other data envelopment analysis models. Resource allocation of inputs and goal setting is demonstrated based on variable returns to scale. Inputs and outputs are measured with precise values. Ma and et al. (2018) proposed Stackelberg and collective data envelopment analysis models for two-stage systems with shared resources. In this paper, they evaluated the efficiency of two-stage systems with shared inputs in Stackelberg competition and cooperation situations. They presented the collective data envelopment analysis model with a two-stage network structure, including external inputs in the second stage. The overall system performance is intuitively demonstrated. The need for such systems arises from the fact that the internal structure of a complex system is not only reflected in sub-stage organizations but also in the allocation of resources, with operational relationships between sub-stages. Sadeghi and Dehnokhalaji (2018) extended two centralized resource allocation methods based on the Lozano and Villa method (2004). The main hypothesis of this research is on decision-making units under a central decision-making unit. It introduces all the targets related to the inputs and outputs of each unit in the next production period. They consider two ideas. The first one was to increase the outputs produced by designing resources and eliminating non-operational inputs as much as possible. Then bring the units to a strong efficient state. The second idea optimized the income and cost functions so that they achieve the best performance. The proposed models examined both constant and variable returns to scale. It then demonstrates that the output targets and the allocation of input resources belong to the production possibility set (PPS). Momeni and et al. (2019) presented centralized data envelopment analysis based on emissions permits under environmental and trade regulations, taking into account the performance of countries. Ding and et. al. (2019) introduced the fixed centralized resources allocation problem for a two-stage network production structure. Specifically for collective two-stage models, they first evaluated the performance of each DMU with a two-stage collective model. Then, they introduced a cost allocation scheme that allows DMUs to operate efficiently under the assumption of CCR. By introducing the concept of maximum satisfaction degree and fairness degree, they proposed a method for obtaining an optimal allocation scheme under centralized control. Kamyab and et al. (2020) proposed a centralized resource allocation model on a two-stage network structure using ratio data envelopment analysis. They applied this model to 13 world commercial banks. Ceasaroni (2020) has proposed an integrated framework for analyzing and determining relationships between technical and centralized resource allocation of cost efficiency, along with output allocation for a number of companies. Furthermore, is delved into the interpretation of technical efficiency measures and their associations with cost analogs. The paper introduced an algorithm for solving nonlinear programming problems related to this subject. For decision-makers, a proper method for computing and comparing a combination of inputs, outputs, and the optimization of multiple units is presented. Tao and et al. (2020) have presented data envelopment analysis based on centralized resource allocation with network flows and the resource allocation profit function. They introduce quantitative analysis using the trade-off between production profits from resource allocation and resource allocation costs in the allocation process. Afsharian and et al. (2021) have reviewed data envelopment analysis approaches with the application of commonly used weights from the perspective of centralized management, where resource allocation costs are described. They determine the optimal resource flows. Madadi and et al. (2022) have proposed a centralized resource allocation model for energy conservation and environmental pollution reduction. This model is designed based on multi-objective programming with the presence of undesirable outputs. The results obtained from the model show that the reduction in overall environmental pollution is proportionally greater than the reduction in total desirable outputs in energy savings. Fang (2022) measured group performance under centralized management. The meta frontier shapes are identified in the decomposition of centralized performance indicators. A new decomposition method, which dominates over the shapes, has been proposed. Chu and et al. (2022) have suggested a healthcare resource allocation method for hospitals based on data envelopment analysis. They have designed a bi-objective model with the first objective being the increase in output targets and the second objective being the allocation of resources proportional to the sizes of the units. To solve this bi-objective model, they have recommended using a trade-off model to obtain resource allocation results. Arocena and et al. (2022) have proposed a directional distance model for efficient resource allocation. A centralized decision-maker oversees all units. The designed model allocates financial aid from higher government layers to municipalities under judicial supervision. The aim of this formula is to inform policymakers about achieving effectiveness, efficiency, and equitable resource utilization. This helps the decision-maker in several ways. First, it allows determining the overall optimal number of financial resources. The municipality needs to cover assumed public tasks and expected needs, so it allows estimating the potential reserves that could be derived from public resources in providing local services. The presented formula is based on the Russell directional distance function with weights. The decision-maker is allowed to simultaneously expand outputs and contract inputs, while facilitating priority setting. Soltanifar and et al. (2022) addressed an important issue of resource allocation efficiency for various operational units. In this research, they introduce a novel approach to resource allocation and target setting. This method utilizes common-weight set and multi-objective optimization. Both of them are compatible with a centralized decision-making character. [Podinovski](https://pubsonline.informs.org/action/doSearch?text1=Podinovski%2C+Victor+V&field1=Contrib) (2022) proposed a resource allocation model for systems in which certain input and output components are shared among decision-making units (DMUs). In other words, for each unit, the corresponding value of those components is unknown, and a general value is defined for all units. Therefore, inputs and outputs are categorized based on the system's conditions. These components are considered with the assumption of a union of independent and shared input indicators, as well as for outputs. Then, he discussed the principles of convexity and scalability under the given conditions and presented an appropriate resource allocation model.

Mohammadi Nejad and et al. (2023) have proposed a model for centralized resource allocation based on data envelopment analysis with managerial feasibility. This model considers the presence of undesirable outputs with the objective of reducing these undesirable outputs. The proposed model is suitable from both environmental and economic perspectives and offers the benefits of resource allocation, target setting, and maximizing overall efficiency. (Models of resource allocation are under the assumption of managerial feasibility and target setting with the presence of undesirable outputs). On the other hand, Lozano and Villa (2023) have suggested a method for sum of fixed output using a multi-objective analysis based on centralized data envelopment analysis. The weighted Chebyshev method is utilized for this purpose. The goal of this approach is to adjust the sum of output objectives as close as possible to the ideal values. The model has been applied to the Tokyo 2020 Olympics. Amirteimoori et al. (2024) introduced a stochastic resource allocation model encompassing random data and undesirable outputs. An illustrative study was conducted in the power industry, involving 21 electricity production and distribution companies over an eight-year period (2011–2019), to compare resource reallocations and their efficiencies. Key findings include: In cases, two companies were decided to be deactivated, reductions in fuel consumption, employees, and net electricity generation are imperative. These reductions are projected to result in a decrease in pollutants. Also, the low price of electricity in Iran has spurred excessive consumption of this product, consequently leading to inefficiencies in numerous companies. Bai and Wang (2024) introduced a Distributed-Optimization with Centralized-Refining (DO-CR) mechanism aimed at enhancing resource allocation efficiency by involving both access points and all devices. The DO-CR mechanism operates in two phases: Initially, it leverages the distributed processing capabilities of all devices, enabling them to optimize their resource allocation schemes using a novel resource reservation and reporting technique. Subsequently, a centralized optimizer constructs a resource trading topology graph based on the individual optimization results and achieves the Pareto optimal solution through a graph-based algorithm. Sakar and Buzat (2024) employed Centralized DEA models to efficiently allocate human resources across various teams within the overall project. Using the software teams from a project undertaken within a large technological defense company as Decision Making Units (DMUs), they identified suitable outputs to gauge effort, work importance, and schedule performance. As for factors pertinent to work importance, these were aggregated using Rank Order Centroid weights based on expert evaluations. Additionally, they devised staffing policies with varying degrees of flexibility applicable in different time periods and recommended the transfer or removal of certain personnel. The findings illustrated that adopting a centralized perspective on human resource allocation can yield significant savings, particularly in the long run when there's greater flexibility in staffing decisions. Madadi et al. (2024) created a model to allocate resources centrally, focusing on environmental technology. Their model deals with unwanted outputs using the weak disposability principle.

**3 Centralized Data Envelopment Analysis**

One of the important research issues from a management perspective is the allocation of ideal resources and setting appropriate targets. The goal is, that systems reach their maximum production potential within a given production possibility set corresponding to the available technologies. Therefore, centralized data envelopment analysis (CDEA) is an approach that addresses this issue. In centralized data envelopment analysis, all inputs and outputs are aggregated under the supervision of a central unit, and then resources allocated based on constraints and the sizes of decision-making units. It is important to note that the objective in centralized data envelopment analysis models is to reduce the total inputs and increase the total outputs. Ultimately, the goal is to maximize the performance of the systems. In data envelopment analysis problems, the linear programming problem is solved for the all of DMUs under evaluation. But, in CDEA, a linear programming problem is simultaneously solved. It reduces the total inputs to be equal or less than the total initial inputs of all decision-making units. Furthermore, the total outputs increase simultaneously or maintain the minimum total production obtained from the total input consumption. So far, centralized resource allocation models based on data envelopment analysis have been presented under the assumptions of constant returns to scale or variable returns to scale. This means that when developing the system, it is assumed that the ratio of changes in the sum of each input index to output index is the same or less or greater than one. In this Section a centralized resource allocation model based on conventional data envelopment analysis is discussed. However, there are some systems in which, for their development or under specific circumstances, the coefficient of changes in all indicators is not necessarily the same. For example, some indicators may be non-changing, some may change proportionally, and others may change with distinct coefficient relative to other indicators. In other words, the coefficient of development of indices has flexibility. Such systems can be exemplified by the educational systems of countries, both at the school and university levels. Assuming that no changes are needed in the number of managements of every educational sector, on the other hand, an increase in prominent teachers and professors or their synergy would result in a proportional increase in the production of elites, graduates, and various projects and inventions. Therefore, in line with such systems, in Section 4, the production technology for the centralized resource allocation model based on DEA has been expanded upon. Corresponding models for these technologies, have been designed. In which the key principle of pseudo-returns to scale (P-RTS) is present. The symbols used in this paper are listed in Table 1.

**Table 1** Symbols and Definition

|  |  |  |  |
| --- | --- | --- | --- |
| **Definition** | **Symbols** | **Definition** | **Symbols** |
| Amount of input *i*th for DMU*j* | $$x\_{ij}$$ | Index corresponding to DMUs | $$j=1,…,n$$ |
| The linear combination vector of DMU $jth$ corresponding to $DMU\_{k}$ |  | Index corresponding to DMUs after resource allocation | $$k=1,…,n$$ |
| The intensity vector of DMU $jth$ corresponding to $DMU\_{k}$ |  | Specified index corresponding to first category inputs | $$i=1,…,t;(i\in I\_{1})$$ |
| Excess slack of the input *i*th | $$s\_{i}^{-}$$ | Specified index corresponding to second category inputs | $$i=t+1,…,p;(i\in I\_{2})$$ |
| Shortfall slack of output *r*th | $$s\_{r}^{+}$$ | Specified index corresponding to third category inputs | $$i=p+1,…,m;(i\in I\_{3})$$ |
| Coefficient of variation of corresponding to the$ i$th | $$θ\_{i}$$ | Specified index corresponding to first category outputs | $$r=1,…,h;(rϵO\_{1})$$ |
| Coefficient corresponding to the changes of third category inputs  | $$t^{'}$$ | Specified index corresponding to second category outputs | $$r=h+1,…,s;(rϵO\_{2})$$ |
| Coefficient corresponding to the changes of second category outputs | $$t^{"}$$ | Amount of output for rth DMU*j* | $$y\_{rj}$$ |

 Lozano and Villa (2004) proposed a centralized data envelopment analysis model with  congruent decision-making units with  input and  output indicators as follows. The presented model allocates resources in two phases at oriented input. In the first phase, the radial oriented input resource allocation model is presented in a centralized manner under variable returns to scale.



Model (1) is a linear programming model with  variables and constraints. Assuming that the optimal value  of Model (1) is obtained, the corresponding slacks for input and output indicators are calculated while preserving optimality in the second phase by solving the model. In the second phase, Model (1) is formulated as follows to maximize the slack variables of surplus, deficiency, while preserving the optimality obtained from the first phase.



 The model of Lozano and Villa (2004) under constant returns to scale is constructed as follows:



 Model (3) is a data envelopment analysis model based on centralized resource allocation under the assumption of constant returns to scale. In this model, the principle of unbounded ray is applied. After solving the model, corresponding vectors  for each decision-making unit $k$ at the new point are obtained. The inputs and outputs at each new point are determined (4), by solving Lozano and Villa's model (2004).



 Models (1) and (2) can be formulated as a multi-objective programming model. By introducing excess variables for the input constraints and deficit variables for the output constraints in model (3), and applying the constant returns to scale principle in Model (5) is derived.



 In the following section, the developed centralized resource allocation model based on data envelopment analysis is presented for cases where the coefficient of changes in the total input and output indicators is not the same. By developing the returns to scale principle and providing various technologies, the proposed models are designed.

**4 CRA development under the Pseudo-Returns to Scale**

In the section, the development centralized resource allocation models are presented. Significantly, there are four main differences between the developed DEA model and the proposed developed CRA model. In developed DEA models solve linear programming (LP) models independently for each decision-making unit. But, in development CRA, only solves a linear programming problem. Simultaneously, it identifies the image of each decision-making unit by applying the coefficient of distinct changes on the indicators. In developed centralized DEA, instead of reducing every input indictor of each decision unit, it reduces the sum of all inputs at once by considering the coefficient of distinct changes among the sum of each indicator. In developed CDEA, instead of increasing production for each decision-making unit, it simultaneously increases or maintains the total of all outputs by applying the condition of the coefficient of distinct changes between the sum of each indicator and setting an appropriate target. In conventional DEA models for example the Charnes-Cooper-Rhodes model, the coefficient of changes is the same for all indicators. However, in the developed DEA model, the changes coefficient of some indicators is assumed a multiple of the development coefficient of other indicators. The conventional data envelopment analysis model defines principles for constructing production technologies and creating the production possibility set. Some of these principles include observability of each decision-making unit, the feasibility principle, convexity principle, constant returns to scale principle, and minimum interpolation principle. In the context of returns to scale, based on the proposed assumptions, if the coefficient of changes in all indicators is equal in the direction of system development, it is referred to as the principle of constant returns to scale (Charnes et al., 1978). If the ratio of changes in input indicators to output indicators is less than one, it is termed as the principle of increasing returns to scale (Seiford and Thrall, 1990). In cases where this ratio of changes in input indicators to output indicators exceeds one, it is called the principle of decreasing returns to scale (Färe and Grosskopf, 1985).

 In this paper, the goal is to optimally allocate centralized resources in systems where the variability of indices must be considered distinctively and flexibly for their development. Accordingly, it is necessary to improve the returns to scale principle and define the production possibility set under the assumption that the variability coefficients of indices are flexible. Thus, in this study, by employing the conventional principles of data envelopment analysis, including the observability of decision-making units, feasibility, convexity, and interpolation, along with introducing a new principle called the "pseudo-returns to scale" principle, the production technology under pseudo-returns to scale (P-RTS) is newly defined. This principle is proposed to enhance the concept of returns to scale. Technology (6) has been derived under such conditions.



 Functions  are defined as changes coefficient in the direction of the development of decision-making units. These functions can be defined based on the goals of organizations for their development. In this study, functions for different constraints have been defined as constant and linear functions passing through the origin. Considering the systems under evaluation, a direct relationship can be assumed between the development coefficients of the indices. In this article, for resource allocation and target setting in organizations, the input and output indicators of organizations are classified into three and two categories, respectively. The first category includes the total input indicators of all DMUs that are either not subject to change or have been assumed to be constant. The coefficient of changes in the second category of total input indicators and the first category of total output indicators is assumed to be the same. The ratio of the development coefficient of the total input indicators in the third category to the total output indicators in the second category is assumed to be equal to or less than one. However, the coefficient of changes in the total indicators of the third category of inputs and the second category of outputs is assumed to be a multiple of the development coefficient of other total indicators. Therefore, if the number of input indicators is considered as the "m" component and the number of output indicators as the "s" component, the index of input indicators corresponding to each category is defined as follows



And the index corresponding to the output indicators is defined as follows.



Assume that:

*  is the development coefficient of the total indicators in the first category of inputs.
*  is the development coefficient of the total indicators in the second category of inputs and the first category of outputs.
*  is the development coefficient of the total indicators in the third category of inputs.
*  the development coefficient of the total indicators in the second category of outputs.

With the change of the appropriate variable, technology can be constructed. The are non-negative variables. The goal of centralized data envelopment analysis is to increase total production while reducing total costs. Therefore, the relationship for the variables  is assumed as . These variables indicate the changes coefficient of a group of input and output indicators concerning other indicators. The range of their changes is different, whether contracting or expanding. Therefore, depending on the defined domain, the production possibility sets changes and correspondingly, various technologies are constructed. In Technology (7), the range of changes is defined within the interval [0,1]. Hence, it is assumed that the coefficient of change in the sum of the third group of input indicators and the sum of the second group of output indicators is less than the coefficient of change in the sum of the other indicators. Therefore, the range of the variables are defined . Note that Technology (7) pertains to centralized resource allocation, derived from Technology (6) through variable change and indicators categorization. This technology is called as developed centralized resource allocation (DCRA) under the pseudo-decreasing returns to scale (P-DRS).



Model 8 is designed based on Technology (7). The resulting model is a non-radial centralized resource allocation model in oriented-input. In this model, the coefficient of development in the total of the third category of inputs and the second category of outputs is multiple from the sum of other categories. In this model, by minimizing ( the coefficients corresponding to the second categories of inputs) and by maximizing slacks corresponding to the total of the output category, appropriate target for the units is obtained.



Model (8) is a multi-objective programming problem. It first minimizes the change coefficients of the inputs, then, based on the optimality of the first part of the objective function, increases the slack corresponding to the outputs. Therefore, if objective function is included  corresponding to the third category of inputs, the value of the coefficient  tends towards the lower bound of its change range. This model is feasible from mathematical perspective. But it is possible that the results of resource allocation using the developed centralized data envelopment analysis model may not be acceptable from a managerial perspective. To address this issue, constraint (h) is added to the problem. Imposing constraint (h) in the model ensures that the total benchmark assigned to the third category of inputs is at most 0.9 of the observed total inputs for the third category. It ensures that for each indicator corresponding to each DMU, a numerical value is allocated based on the problem's constraints. Therefore, sometimes, it is necessary to add one or more constraints to the problem in case there is a limitation on the indicators corresponding to each unit. For example, if the manager does not allow to change the third input indicator of the second unit should not exceed 5 units according to the system's conditions, the added constraint becomes of the form .

The objective function of model (8) is a multi-objective linear function of minimization type. The optimal value of the objective function is equal to .

Top of Form

Constraint category (a) pertains to finding a corresponding point for the indicators in the first category by centralized allocation (assuming the sum of one indicator among all units does not require any changes in proportion to the sum of the indicators from other categories). Constraint category (b) relates to the allocation of resources for the indicators in the second category of inputs (The coefficient of changes in the total of this category of input indicators is equal to the coefficient of changes in the total of the first category of output indicators). Constraint category (c) relates to the allocation of resources for the indicators in the third category of inputs. With the coefficient of changes $t^{'}$, which is positive and equal or less than one. It allows to change in the sum of inputs across all units, based on the $θ\_{i}$ variable as non-radial. Constraint category (d) pertains to target setting for the first category of output indicators to reach the desired level. Constraint category (e) concerns setting the target for the second category of output indicators. The coefficient of changes is *t"* that is equal or less than one (The coefficient of changes in the total of this category of output indicators is greater than the coefficient of changes associated with constraint (c), but that is a multiple of the changes coefficient of constraints category (b) and (d)). The constraint (f) represents the convex combination related to the first category of inputs. Constraint category (g) is related to the intensity vector coefficients, indicating the principle of P-DRS The purpose of constraint (h) is to allocate costs to the units while maintaining a maximum of 0.9 of the observed total costs related to the third category of inputs. And, the constraint (I) describes the domain of changes coefficients.

 Now, the production technology for centralized resource allocation is constructed for the case when the change coefficient for the sum of output indicators in the second category and input indicators in the third category is regarded as a multiple of the development coefficient for the sum of other indicators. Note, these coefficients are assumed to be non-negative variables. It is also assumed that the sum of the indicators in the first category across all units does not require changes in proportion to the other indicators.



Technology (9) is called to as developed centralized resource allocation under the pseudo-constant returns to scale (P-CRS) principle. The corresponding model with (9) is designed as (10).



By solving Model (10) without imposing constraints (g) and (h), the total input costs do not decrease for some indicators. Additionally, the total of production obtained from the observed data is not achieved. Therefore, to address this issue and interact with the manager for solving it, constraints (g) and (h) are applied to the model, resulting in acceptable outcomes. Limitations can be applied to certain indicators through interaction with the manager, that bring the results closer to reality and the manager's goal. In this model, by considering constraints on input and output indicators, the results are re-evaluated, and appropriate targets are achieved.

Here, the production technology for centralized resource allocation is obtained when the coefficient of changes in the third category of total input indicators and the second category of total output indicators is greater than other indicators, assuming the first category of inputs is fixed. In this technology, the variables  represent the coefficient of changes in the third category of total input indicators and the second category of total output indicators, respectively. Their range are considered as . The corresponding technology is constructed as follows:



This technology is called developed centralized resource allocation under the pseudo-increasing returns to scale (P-IRS). Then, the optimal value of the objective function is reduced by increasing the slack variables $s\_{r}^{+}$. The non-radial model corresponding to technology (11) is designed as oriented input.



In model (12), the second and third category constraints of input indicators are allowed to change relative to $θ\_{i}$ to reach the desired level. By solving model (12), without constraint (h) and (I), the total resources allocated to this category of input indices exceed the total initial consumption. Since the goal in centralized resource allocation is resource savings, the constraint  is considered for the all inputs. Note, that this model is mathematically solvable, but the resulting goal may not be acceptable from a managerial perspective. So, by interacting with the manager and considering the constraints on each output index, constraint can be added to the problem constraints.

**Proposition 1** The efficiency of targets obtained from resource allocation corresponding to each of model (8) on the defined production possibility set improves or does not worsen compared to the efficiency of observable DMUs.

**Proof** Let's assume that for every  target setting obtained from solving model (8),  is explicitly determined. Using a proof by contradiction, suppose that the point  is not efficient or its efficiency does not improve. Therefore, by solving model (8), we obtain a vector that satisfies in. the point of target setting corresponding to is defined as follows:



So, at least one of the input or output components must exhibit strict inequality. Let's assume that on a specific input component , which can be from any of the second or third categories of inputs, strict inequality holds. Thus, we can write 

So, the optimal solution obtained from model (8) as oriented-input corresponding to and the vector  instead of the assumed optimum is equal to  and for the other input components, we have:



As a result, a feasible solution with a lower value for the objective function is obtained for model (8). On the other hand, it is possible that if at least one specific component like belongs to the first category of outputs, strict inequality is established such that **

Alternatively, if the component belongs to the second category of output indicators, strict inequalities ** are established. Consequently, based on Model (8), corresponding to  vector  is directed towards a feasible solution So, the following inequalities corresponding to each category of constraints related to the outputs are resulted:



which is better than the previous optimum. Therefore, we encounter a contradiction. Thus, the target settings corresponding to each DMU obtained from solving Model (8) may dominate the assumed observable data of each DMU. Consequently, the unit's efficiency values do not worsen with the data obtained from resource allocation. □

**Note 1** The proposition is similarly provable for models 10 and 12.

**Result** **1** From proposition 1, is concluded that the average of the total efficiency values of the decision-making units with the target setting obtained from solving Model (8) is greater than the average of the total efficiency values of the observable DMUs.

**5 Example**

Let's consider an example where we are evaluating the efficiency of multiple systems and then allocating centralized resources under a single supervisory unit. Centralized resource allocation (CRA) is one of the recommendations aimed at improving system performance. The objective of CRA development is to minimize the total input costs of systems and enhance system performance. Consequently, it seeks to maximize or maintain the total production from the allocated resources. In this section, we examine the results obtained from centralized resource allocation under the P-RTS principles using a numerical example. Suppose, we have 20 decision-making units with three input and two output indicators. It is assumed that, there is no need for changes in the first group's total inputs relative to the sum of the other indicators. The total inputs of the second indicator and the total outputs of the first indicator change in the same proportion for each DMU. The coefficient of development in the total inputs of the third indicator and the total outputs of the second indicator is a multiple from the coefficient of changes in the other indicators for each DMU. In Table 2, numerical data related to each decision-making unit and each index is presented. The sum of each input and output index for the units is also shown in the last row of the table. In Tables 3, 4, and 5, the targets obtained from solving the proposed models 8, 10, and 12, respectively, are displayed.

**Table 2** Data set

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Efficiency on $T\_{IRS}$  | Efficiency on $T\_{DRS}$  | Efficiency on $T\_{CRS}$  | Output2 | Output1 | Input3 | Input2 | Input1 | DMUs |
| 0.8074 | 0.7991 | 0.7991 | 8 | 65 | 5 | 3 | 10 | **DMU1** |
| 0.6177 | 0.6177 | 0.6177 | 10 | 53 | 6 | 3 | 12 | **DMU2** |
| 0.6088 | 0.5598 | 0.5598 | 5 | 61 | 4 | 4 | 17 | **DMU3** |
| 0.6352 | 0.6352 | 0.6352 | 6 | 49 | 6 | 2 | 8 | **DMU4** |
| 0.8185 | 0.8185 | 1 | 12 | 74 | 8 | 2 | 10 | **DMU5** |
| 1 | 0.7628 | 0.7628 | 5 | 50 | 4 | 3 | 6 | **DMU6** |
| 0.5433 | 0.4920 | 0.4920 | 7 | 39 | 5 | 4 | 11 | **DMU7** |
| 0.3549 | 0.3210 | 0.3210 | 6 | 45 | 7 | 5 | 12 | **DMU8** |
| 0.4727 | 0.4727 | 1 | 15 | 53 | 10 | 3 | 15 | **DMU9** |
| 1 | 1 | 1 | 6 | 72 | 5 | 5 | 5 | **DMU10** |
| 0.6380 | 0.6380 | 0.6380 | 8 | 38 | 6 | 2 | 7 | **DMU11** |
| 1 | 1 | 1 | 6 | 60 | 7 | 1 | 11 | **DMU12** |
| 0.5438 | 0.4857 | 0.4857 | 7 | 52 | 5 | 3 | 18 | **DMU13** |
| 1 | 1 | 1 | 14 | 35 | 8 | 2 | 14 | **DMU14** |
| 1 | 1 | 1 | 10 | 71 | 6 | 2 | 5 | **DMU15** |
| 1 | 1 | 1 | 13 | 59 | 9 | 1 | 4 | **DMU16** |
| 1 | 0.7098 | 0.7098 | 6 | 48 | 4 | 3 | 9 | **DMU17** |
| 1 | 0.6158 | 0.6158 | 5 | 37 | 5 | 2 | 8 | **DMU18** |
| 1 | 1 | 1 | 7 | 77 | 3 | 4 | 14 | **DMU19** |
| 1 | 1 | 1 | 11 | 39 | 6 | 3 | 10 | **DMU20** |
| - | - | - | 167 | 1077 | 119 | 57 | 206 | **Summation** |

 In the leftmost column, the DMUs numbers are listed. The next three columns display the observed data corresponding to the input indicators for each DMU. The fifth and sixth columns show the observed data related to the output indicators.Top of Form

 The seventh, eighth, and ninth columns of Table 2 display the efficiency values obtained from solving the models related to technologies under CRS, DRS, and IRS, respectively. In Table 3, the targets obtained from solving Model 8 are provided. The domain of changes in the total of the third input index of the decision-making units and the total of the second output index are equal or less than one. The sum of the first input index of the units is assumed to have no coefficient of change.

**Table 3** Targets obtained from solving Model 8

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Efficiency | t" | t' | Output2 | Output1 | Input3 | Input2 | Input1 | DMUs |
| 0.9673 | 1 | 0.5 | 7.5 | 53.25 | 4.5 | 1.5 | 7.7424 | **DMU1** |
| 0.9670 | 1 | 0.5 | 9 | 63.9 | 5.4 | 1.8 | 9.1326 | **DMU2** |
| 0.9873 | 1 | 0.5 | 6 | 42.6 | 3.6 | 1.2 | 5.8496 | **DMU3** |
| 1 | 1 | 0.5 | 9 | 63.9 | 5.4 | 1.8 | 8.2870 | **DMU4** |
| 1 | 1 | 0.5 | 11.0211 | 65.8112 | 7.2 | 1.5708 | 8.2870 | **DMU5** |
| 1 | 1 | 0.5 | 5.7210 | 35.9746 | 3.6 | 0.9210 | 8.2870 | **DMU6** |
| 0.9695 | 1 | 0.5 | 7.0826 | 43.3361 | 4.5 | 1.0826 | 8.2870 | **DMU7** |
| 0.9209 | 1 | 0.5 | 9.8057 | 58.0592 | 6.3 | 1.4057 | 8.2870 | **DMU8** |
| 1 | 1 | 0.5 | 13 | 59 | 9 | 1 | 4 | **DMU9** |
| 0.7802 | 1 | 0.5 | 7.5 | 53.25 | 4.5 | 1.5 | 12.5309 | **DMU10** |
| 1 | 1 | 0.5 | 4.9983 | 47.7756 | 5.4 | 0.8584 | 12.5309 | **DMU11** |
| 1 | 1 | 0.5 | 10.3 | 69.8 | 6.3 | 1.9 | 12.5309 | **DMU12** |
| 0.7802 | 1 | 0.5 | 7.5 | 53.25 | 4.5 | 1.5 | 12.5309 | **DMU13** |
| 0.8889 | 1 | 0.5 | 11.2 | 66.2 | 7.2 | 1.6 | 12.5309 | **DMU14** |
| 0.8342 | 1 | 0.5 | 9 | 63.9 | 5.4 | 1.8 | 12.5309 | **DMU15** |
| 0.7778 | 1 | 0.5 | 12.1 | 62.6 | 8.1 | 1.3 | 12.5309 | **DMU16** |
| 0.7262 | 1 | 0.5 | 6 | 42.6 | 3.6 | 1.2 | 12.5309 | **DMU17** |
| 0.7471 | 1 | 0.5 | 7.3272 | 49.1457 | 4.5 | 1.3272 | 12.5309 | **DMU18** |
| 0.6723 | 1 | 0.5 | 4.5 | 31.95 | 2.7 | 0.9 | 12.5309 | **DMU19** |
| 0.7731 | 1 | 0.5 | 8.4441 | 50.6976 | 5.4 | 1.2441 | 12.5309 | **DMU20** |
|  | - | - | 167 | 1077 | 107.1 | 27.4098 | 206.0001 | **Summation** |

 Table 3 demonstrates that by solving Model 8, different targets have been obtained for the decision-making units. The efficiency values for each DMU have been calculated. Some of units have reached the level of efficiency. The coefficient of change for the third category of input indices is equal to 0.5 times the development coefficient of other indices. And all other indicators have changed proportionally to each other except the first category of input indicators. This means that if we reduce the costs associated with the third category of input indicators by implementing Model 8, in addition reducing the total input costs compared to the observed data. The efficiency of the DMUs changes and improves in most of them. In Table 4, the targets are derived from solving Model 10. The change coefficients for the sum of the third input indicators and the sum of the second output indicators of the decision-making units are different from the change coefficients for other indicators.

**Table 4** Targets obtained from solving Model 10

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Efficiency | t" | t' | Output2 | Output1 | Input3 | Input2 | Input1 | DMUs |
| 1 | 0.97 | 0.49 | 8.6145 | 65 | 5 | 2.0884 | 11.2058 | **DMU1** |
| 1 | 0.97 | 0.49 | 10 | 53 | 6 | 1.6821 | 5.8345 | **DMU2** |
| 1 | 0.97 | 0.49 | 7.3253 | 61 | 4 | 2.3213 | 5.8345 | **DMU3** |
| 1 | 0.97 | 0.49 | 6 | 49 | 6 | 0.8307 | 8.0923 | **DMU4** |
| 1 | 0.97 | 0.49 | 12 | 74 | 8 | 1.7599 | 6.9704 | **DMU5** |
| 1 | 0.97 | 0.49 | 6.7952 | 50 | 4 | 1.5261 | 11.4248 | **DMU6** |
| 0.6935 | 0.97 | 0.49 | 7 | 39 | 5 | 0.7793 | 10.7433 | **DMU7** |
| 1 | 0.97 | 0.49 | 6 | 45 | 5.6972 | 0.7535 | 11.4248 | **DMU8** |
| 1 | 0.97 | 0.49 | 15 | 61.5909 | 10 | 1.3636 | 11.2058 | **DMU9** |
| 0.4910 | 0.97 | 0.49 | 6 | 72 | 5 | 5 | 11.2058 | **DMU10** |
| 1 | 0.97 | 0.49 | 8 | 38 | 5.6855 | 0.6432 | 11.2058 | **DMU11** |
| 1 | 0.97 | 0.49 | 6 | 60 | 7 | 1 | 11.2058 | **DMU12** |
| 0.8213 | 0.97 | 0.49 | 7 | 52 | 5 | 1.2834 | 11.2058 | **DMU13** |
| 1 | 0.97 | 0.49 | 14 | 35 | 8 | 2 | 11.2058 | **DMU14** |
| 1 | 0.97 | 0.49 | 10 | 71 | 6 | 2 | 11.2058 | **DMU15** |
| 1 | 0.97 | 0.49 | 13 | 59 | 9 | 1 | 11.2058 | **DMU16** |
| 1 | 0.97 | 0.49 | 6.6988 | 48 | 4 | 1.3815 | 11.2058 | **DMU17** |
| 0.7459 | 0.97 | 0.49 | 5 | 37 | 4.7042 | 0.6197 | 11.2058 | **DMU18** |
| 1 | 0.97 | 0.49 | 7 | 77 | 3 | 4 | 11.2058 | **DMU19** |
| 0.6602 | 0.97 | 0.49 | 11 | 39 | 6 | 3 | 11.2058 | **DMU20** |
|  |  |  | 172.4337 | 1085.59 | 117.0869 | 35.0327 | 205.9999 | **Summation** |

 Table 4 indicates that, by solving model 10, the targets attained in some DMUs have reached the efficiency frontier. Furthermore, the efficiency values of these units have become equal to one. This implies that the targets are positioned on the boundary of the corresponding production possibility set under pseudo-constant returns to scale (P-CRS) technology. In the last row of Table 5, the total inputs and total outputs are provided. The results indicate that by executing Model 10 under the proposed technology, the total input costs have decreased, while the total production has increased. Additionally, the change coefficient for the sum of the third category of input indicators is 0.49, and the change coefficient for the sum of the second category of output indicators is 0.97 compared to the changes in other indicators. In Table 5, the targets obtained from solving Model 12 are presented. The range of change coefficients for the sum of the third category of input indicators and the sum of the second category of output indicators has been set equal to or greater than one.

**Table 5** Targets obtained from solving Model 12

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Efficiency | t" | t' | Output2 | Output1 | Input3 | Input2 | Input1 | DMUs |
| 1 | 1.023 | 1 | 7.6 | 61.75 | 5 | 2.4521 | 18 | **DMU1** |
| 1 | 1.023 | 1 | 9.5 | 68.5 | 6 | 1.9444 | 18 | **DMU2** |
| 1 | 1.023 | 1 | 6.0452 | 57.95 | 4 | 3.0151 | 4 | **DMU3** |
| 1 | 1.023 | 1 | 6.0053 | 46.55 | 5.5026 | 1.8325 | 4 | **DMU4** |
| 1 | 1.023 | 1 | 11.4 | 70.3 | 7.2234 | 1.7688 | 4 | **DMU5** |
| 1 | 1.023 | 1 | 5 | 50 | 4 | 3 | 4 | **DMU6** |
| 1 | 1.023 | 1 | 6.65 | 51.85 | 5 | 2.275 | 4 | **DMU7** |
| 0.9334 | 1.023 | 1 | 6 | 60 | 7 | 1 | 6.7992 | **DMU8** |
| 1 | 1.023 | 1 | 14.25 | 64.6731 | 9.8654 | 1.0962 | 13.7971 | **DMU9** |
| 0.5969 | 1.023 | 1 | 6 | 72 | 5 | 5 | 13.7971 | **DMU10** |
| 1 | 1.023 | 1 | 7.6 | 57.2 | 5.8 | 1.8444 | 13.7971 | **DMU11** |
| 1 | 1.023 | 1 | 6 | 60 | 7 | 1 | 4 | **DMU12** |
| 1 | 1.023 | 1 | 6.65 | 51.85 | 5 | 2.275 | 4 | **DMU13** |
| 1 | 1.023 | 1 | 13.3 | 44.8 | 8 | 1.7667 | 13.7971 | **DMU14** |
| 1 | 1.023 | 1 | 10 | 71 | 6 | 2 | 13.2431 | **DMU15** |
| 1 | 1.023 | 1 | 13 | 59 | 9 | 1 | 13.2431 | **DMU16** |
| 0.9084 | 1.023 | 1 | 5.7 | 52.65 | 4 | 3 | 13.2431 | **DMU17** |
| 1 | 1.023 | 1 | 5 | 37 | 5 | 2 | 13.2431 | **DMU18** |
| 1 | 1.023 | 1 | 7 | 77 | 3 | 4 | 13.2431 | **DMU19** |
| 1 | 1.023 | 1 | 10.45 | 63.8 | 6 | 2.3 | 13.7971 | **DMU20** |
|  |  |  | 163.1505 | 1177.873 | 117.3914 | 44.5701 | 206 | **Summation** |

 Table 5 illustrates the targets obtained from solving Model 12. As observed, input costs have decreased, while the total production has increased except in total second group of output indicator. The sum of second category of output is decreased almost 2 percent. However, significant changes of 102% in the total of the second category of outputs have been implemented. The efficiency values of these targets have been calculated under the proposed technology, showing improvements compared to before resource allocation. And some units have reached efficiency frontier. According to the results given in tables 3, 4, and 5, total consumption costs have noticeably decreased, while in some cases, they remain at the same level as the initial consumption. Total production has increased in some indicators and remained unchanged in others. Additionally, significant changes in coefficients have been applied to some indicators. Therefore, considering the proposed technologies and applying different coefficients of changes in the indicators leads to acceptable and desirable results. With centralized resource allocation based on data envelopment analysis in all three proposed models, the average efficiency of decision-making units relative to the average efficiency of each decision-making unit with observed initial data is higher. Note that the with managerial decisions, constraints can be imposed on the percentage of consumption resources and resulting outputs for each DMU. Table 6 presents the targets obtained from solving Model 5, which is constructed based on the two-phase model by Lozano and Villa (2004) under CRS technology.

**Table 6** Targets obtained from solving Model 5

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Output2 | Output1 | Input3 | Input2 | Input1 | DMUs |
| 10 | 71 | 6 | 2 | 5 | **DMU1** |
| 10 | 71 | 6 | 2 | 5 | **DMU2** |
| 6 | 48 | 4 | 3 | 9 | **DMU3** |
| 10 | 71 | 6 | 2 | 5 | **DMU4** |
| 10 | 71 | 6 | 2 | 5 | **DMU5** |
| 10 | 71 | 6 | 2 | 5 | **DMU6** |
| 10 | 71 | 6 | 2 | 5 | **DMU7** |
| 10 | 71 | 6 | 2 | 5 | **DMU8** |
| 10 | 71 | 6 | 2 | 5 | **DMU9** |
| 10 | 71 | 6 | 2 | 5 | **DMU10** |
| 6 | 48 | 4 | 3 | 9 | **DMU11** |
| 10 | 71 | 6 | 2 | 5 | **DMU12** |
| 6 | 48 | 4 | 3 | 9 | **DMU13** |
| 10 | 71 | 6 | 2 | 5 | **DMU14** |
| 6 | 48 | 4 | 3 | 9 | **DMU15** |
| 8.3266 | 61.3778 | 5.16 | 2.4184 | 6.6734 | **DMU16** |
| 5 | 37 | 5 | 2 | 8 | **DMU17** |
| 5.6734 | 50.4685 | 4.33 | 2.6734 | 10.0203 | **DMU18** |
| 7 | 77 | 3 | 4 | 14 | **DMU19** |
| 7 | 77 | 3 | 4 | 14 | **DMU20** |
| 167 | 1275.85 | 102.49 | 49.09 | 143.69 | **Summation** |

 By solving Model 5, six targets were obtained for the DMUs. In the last row of Table 6, the sum of allocated consumable resources corresponding to each input index is provided. The total production corresponding to each output index has been calculated for all DMUs.

 Figure 1 presents a comparative diagram of the percentage of total resource savings and the percentage increase in production (about every indicator) for the proposed centralized resource allocation models and model (5) compared to the observed data of DMUs.

**Fig1** Comparative diagram of resource savings and production increases among models 8, 10, 12, and 5

 The coefficient of changes in the total of some indices is assumed a multiple of the sum other indicators. Under these conditions, technologies corresponding to centralized resource allocation are developed. The results obtained from resource allocation with the implementation of proposed models are as follows:

* By implementing Model 8, it resulted 0%, 52% and 10% resource saving in each input indicator respectively. It has been able to achieve the same total primary production with these resource savings.
* Results from Model 10 show that, in total, resources saving exist or consumable costs have decreased based on the allocated resources. By implementing Model 10, it resulted 0%, 39% and 0.8% resource saving in each input indicator respectively, increase production 0.8% and 3.3% respectively for every output indicator.
* By implementing Model 12, The results related to resource savings and production increases are as follows: 0%, 22% and 1.4% resource saving in each input indicator and 9.4% and -2% increase in production (in every output indictor) compared to the initial data.
* By solving model 5, resource saving was 30%, 13.8% and 13.8% respectively for each input index and production increase was 18% and 0% respectively for each output index.

Note, assuming that the total of the first category of input indices does not necessarily need to change. Due to, the percentage of resource savings in the first input indicator is zero. Therefore, by implementing proposed models such as Models 8 and 10, the results are achieved with approximately 20% resource savings for the all of DMUs. And it has the same amount of initial production.

**6 Results and Suggestions**

In this research, by considering the proposed production possibility sets in the centralized resource allocation based on data envelopment analysis and designing models corresponding to each technology, this approach is yielded various outcomes, which were mathematically feasible. However, from a managerial standpoint, it's essential to engage with managers to ensure proper resource allocation to each decision-making unit. Note that better results can be achieved by considering the distinct coefficient changes in the sum of certain indicators. These models were solved using GAMS software, resulting in increased resource savings and higher production levels. The goal of presenting the proposed models in this paper is to provide a novel approach to the development of systems and centralized resource allocation based on data envelopment analysis. By considering the coefficient of distinct changes in the total indices and setting suitable and efficient targets, or close to efficiency, in the corresponding production possibility set. It also focuses on the proper allocation of resources to prevent the wastage of resources in systems in proportion to the system's needs and size, along with managerial decision-making. In summary, the average efficiency of DMUs with the targets obtained from the proposed models is higher compared to observable units.

 For further research, the proposed idea can be applied to different types of data, including fuzzy, ratio data, random data, as well as to systems with undesirable inputs and outputs. It can also be applied to systems that have a serial or parallel network structure with independent inputs and outputs.

**Compliance with Ethical Standards:**

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