**Value Efficiency Analysis in different technologies**

**Abstract.** One approximation to apply superiority and preference information in Data Envelopment Analysis (DEA) is to use value efficiency approach. The purpose of calculating value efficiency approach is to calculate increase in outputs and reduce in inputs to achieve value function frontier that passes most preferred solution (MPS) point. Note that value function is an unknown function and we can use linear approximation for the approximation of this function and the new frontier will replace the real frontier. In this paper, directional distance function is used to calculate value efficiency. Thus, different values of value efficiency are achieved by selecting various directions. In the following, the above models are used to assess the value efficiency of bank branches by applying the comments of managers and we will see that without the application of weight restrictions, we can apply the comments of managers for a proper assessment.

*Keywords and Phrases:* Data envelopment analysis; Value efficiency; Efficiency analysis; Directional Distance Function.

**1. Introduction:**

Data envelopment analysis is a technique for the assessment of the efficiency of a set of decision-making units (DMU). This method was introduced by Charnes et al. [1] and did not consider any superior information for the units and these models were called free value. Various methods were presented for the application of superiority information in data envelopment analysis. One of these methods was weight restriction. Considering that traditional models of DEA ignore the value of some inputs and outputs in the calculation of efficiency, Dyson and Thanassoulis [5] and Wong and Beasley [17] developed the above methods. Other methods are presented in papers [5, 15, 16, 17, 2].

 Halme, Joro, Korhonen, Salo and Wallenius [8] presented a method for the application of superiority information in efficiency analysis. To calculate DEA, they compared the unit under assessment with an unknown function which is called value function and is quasi -convex and quasi-concave respectively based on inputs and outputs. They introduced the point where value function is at its maximum as MPS which is a virtual or real unit on efficiency frontier. The purpose of value efficiency is to assess the amount the unit under assessment should decrease and increase its inputs and outputs respectively to achieve value function frontier. Regarding that the function and its frontier are unknown; various methods exist for the approximation of this frontier. One method is to define a zone including inputs and outputs vectors having less superiority than MPS and a linear approximation border is constructed. The ratio the inputs and outputs should respectively decrease and increase to achieve approximation border is called value efficiency value which is of a technique efficiency kind. In the following, the projects done by Zohrehbandian [19] with the introduction of a MOLP structure for DEA models whose target function define input and output variables with regard to specifications of production are presented. He used Zionts-Wallenias [18] Method to obtain value efficiency approach (VEA) value and achieve MPS. The achieved weights are determined with regard to the value structure of inputs and outputs. In the following, the projects done by Joro et al. [11] presented an interactive method that search line is based on the radius that go from the origin and then under the assessment unit. It should consider a point with a similar value of MPS, but the presented method was not appropriate in terms of calculation and one problem should be solved for each DMU. Korhonen and Syrjanen [12] presented the concept of Value Efficiency Analysis (VEA). Without any discussion about approximation of value function. They showed that with more restricted assumptions in the interval of value function, the value of VEA and suitable value information can be determined. In the following, you will see that if the assumption of convex production possibility set is not met, primary approximations cannot be used for the calculation of VEA. Thus, the development of the above models for Free Disposable Hull (FDH) model [4] models use convex cones [13] to approximate value function border regarding that production possibility set in these models is not convex. Frontier approximation should be in a way in which the value of value efficiency is greater than technical efficiency. Consider the optimistic mode; the rest of this paper is organized as below. In the second section, the concepts of directional distance function, the value efficiency of models with constant scale return, and FDH variable and model are introduced. In the third section, previous models are developed based on directional distance function. In section four, the presented approximation is applied for the assessment of 25 branches of a bank in Finland as presented in [14]. In section five, the results are compared and the results are presented.

**2. Concepts**

At first, propose value efficiency analysis concept and calculus value efficiency analysis score.

**2.1. Value Efficiency Analysis:** Assume the set of n Decision-Making Units (DMUs) $N=\left\{1,2,…,n\right\} $and each unit consumes m inputs for the production of p outputs. The input and output vectors for the DMU are as $x\in R\_{+}^{m}$ , $y\in R\_{+}^{p}$. The production possibility set is introduced as follows.

$T=\left\{y can be produced from x \right\}\in R\_{+}^{m+p}$,

**Definition 2.1.1.** Point $\left(y^{\*},x^{\*}\right)\in T$ is efficient if and only if there does not exit another $\left(y,x\right)\in T$ such that $x\leq x^{\*}$ , $y^{\*}\leq y$ and $\left(y^{\*},x^{\*}\right)\ne \left(y,x\right).$ Thus it is inefficient and it is possible that it is on inefficiency frontier which is called weak efficiency or it is completely inside the production possibility set.

**Definition 2.1.2.** Point $\left(y^{\*},x^{\*}\right)\in T$ is weak efficiency if and only if there does not exit another $\left(y,x\right)\in T$ such that $<x^{\*}$ , $y^{\*}<y$.

**Definition 2.1.3.** The function$ g:R^{m+p}\rightarrow R$ is called a value function if it has the following specifications [3].

a) $g\left(y^{\*},x^{\*}\right)>g\left(y,x\right) $and if $\left(y^{\*},x^{\*}\right)$ dominates $\left(y,x\right)$ points.

b) $g\left(y^{\*},x^{\*}\right)>g\left(y,x\right) $and if $\left(y^{\*},x^{\*}\right)$ is superior to $\left(y,x\right)$.

c) $g\left(y^{\*},x^{\*}\right)>g\left(y,x\right) $and if $\left(y^{\*},x^{\*}\right)$ is the minimum superiority of $\left(y,x\right)$. And the set DM s preferences as follows. $P=\left\{\left(y,x\right)\_{r}>\left(y,x\right)\_{s}, r,s\in \left\{1,…,n\right\}\right\}$

i.e., if unit $\left(y,x\right)\_{i}$ dominates$ unit \left(y,x\right)\_{j}$, then,$\left(\left(y,x\right)\_{i},\left(y,x\right)\_{j}\right)\in P$. In the following we use the symbol $≻ $to indicate relationship “is preferred to”. Function $g$ is strictly increasing drastically respectively based on output and input in $g\in R\_{+}^{m+p}$. The DM’s value function is, moreover, assumed to be pseudo-concave for outputs and pseudo-convex for inputs, but only its functional form is specified. In addition, for easier $ f\left(y, -x\right)=g\left(y,x\right)$ that f is drastically concave and convex respectively based on output and input.

Assume that$ \left(x\_{j},y\_{j}\right) $is the input and output vectors and the jth DMU and the ith value of input vector component and the kth output vector component are respectively as $x\_{ij}$, $y\_{kj}$, the purpose of VEA is the assessment of efficiency of each unit with regard to the border of value function which passes MPS. But regarding that value function is unknown, we can apply various approximations for its frontier and these functions present all the possible tangents for an unknown function. Now, we will describe the concept of VEA with an example. Consider Table 1 and seven DMUs with an input and output. The applied technology of return is a non-increasing scale.

Table1: The data set of a numerical example

Unit Input Output1 Output2

 A 1 1 10

 B 1 2 9

 C 1 3 7

 D 1 3.4 4

 E 1 4.6 2

 F 1 2 6.5

 G 1 2.6 4.5

Production possibility set is specified in figure 1. Note that the input of all units is one. For example, assume that we want to obtain some VEA of G unit which is an inefficient unit. The value of technical efficiency of this unit is $\frac{\left|OG\right|}{\left|OG^{,}\right|}=0.77 $equal to the Distance of the intended point from the origin to the efficiency frontier.

 

 Figure 1: An illustration of value efficiency analysis of unit G.

The ratio of these two distances is the radial or technical efficiency. $\left|OG^{,}\right| $is the distance between G and efficiency frontier if the G unit selects its MPS according to the figure in a specific point between B and C, assume that this MPS maximizes value function. Regarding that f function is unknown and that MPS is completely superior to B and C, the value efficiency of VEA from G unit equals to ratio $\frac{\left|OG\right|}{\left|OG^{3}\right|}=0.65$, with regard to the hypothetical function in Figure 1. We ourselves assumes this function, if the function is unknown, we should approximate it. To this end, the approximation frontier should include all possible tangents from a false concave function which achieves its maximum amount in MPS. In this mode, the slope or tangent of is uniquely determined as a line segment passing B and C. To determine efficiency value, we should measure the distance of G unit from $G^{2}$ point on the approximation frontier. The value efficiency value is determined as $\frac{\left|OG\right|}{\left|OG^{2}\right|}=0.75$ which is a greater value compared with its real value. If MPS is the extreme point B, approximation frontier is not unique, because Infinity line passes of B. In all cases, our approach to approximating the true Value Efficiency always gives the most optimistic estimate for it. In the sequel we use the term Value Efficiency when we refer to the approximation of the true Value Efficiency. If MPS is a combination of the existing units, suppose $\left(λ\_{1}^{\*}, λ\_{2}^{\*},…,λ\_{n}^{\*} \right)$ is optimal solution of BCC model. we obtain optimal solutions BCC model and MPS is presented as follows.

$y\_{k}^{\*}=\sum\_{j=1}^{n}λ\_{j}^{\*}y\_{kj}$ $k=1,…,P.$

$x\_{i}^{\*}=\sum\_{j=1}^{n}λ\_{j}^{\*}x\_{ij}$ $i=1,…,m.$

$\sum\_{j=1}^{n}λ\_{j}^{\*}=1.$

**2.2 Directional Distance Function**

Consider n decision-making units that consume m inputs ($x\_{ij}$ , $i=1,..,m, j=1,…,n ) $in order to produce s outputs ($y\_{rj}$ ,$ r=1,..,s, j=1,…,n )$. $λ\_{j} $, $j=1,…, n,$ are intensity variables. Assuming the observed units, the directional distance function is introduced as follows.

$β\_{o}^{\*}=Max $ $ β\_{o}$

$s.t.$ $\sum\_{j=1}^{n}λ\_{j}x\_{ij}\leq x\_{i}^{o}-β\_{o}g\_{io}^{-} $, $i=1,…,m,$

 $\sum\_{j=1}^{n}λ\_{j}y\_{kj}\leq y\_{k}^{o}+β\_{o}g\_{ko}^{+} $, $k=1,…,p,$ (1)

 $\sum\_{j=1}^{n}λ\_{j}=1$, $λ\_{j}\geq 0$,

Adding slack and extra variables, the above model is presented as follows.

$β\_{o}^{\*}=Max $ $ β\_{o}$

$s.t.$ $\sum\_{j=1}^{n}λ\_{j}x\_{ij}+s\_{i}^{-}=x\_{i}^{o}-β\_{o}g\_{io}^{-} $, $i=1,…,m,$

 $\sum\_{j=1}^{n}λ\_{j}y\_{kj}- s\_{k}^{+}=y\_{k}^{o}+β\_{o}g\_{ko}^{+} $, $k=1,…,p,$ (2)

 $\sum\_{j=1}^{n}λ\_{j}=1$, $λ\_{j}\geq 0$, $j=1,…, n.$

 Note that directional distance function assumes a direction vector as

$g=\left(-g^{-},g^{+}\right)\ne 0\_{m+s}$ , $g^{-}\in R^{m}$, $g^{+}\in R^{s}$ and each unit under assessment as weak efficiency frontier. Vector g can be constant. In the special case, we can put$ g\_{o}=\left(0 , y\_{o}\right)$, $g\_{o}=\left(-x\_{o} , 0\right)$, $g\_{o}=\left(-x\_{o} , y\_{o}\right)$. We can put vector g as$ g\_{o}=\left(-\left(x\_{o}-\overline{x}\right) , \left(\overbar{y}-y\_{o}\right)\right)$ where the ith component of Minimum$ \overline{x}$ is the input value of ith input in input data set and the rth component of$ \overbar{y} $is the max rth output value in the data set. Note that if $β\_{o}^{\*}=0$ under evaluation Units are efficient and if $β\_{o}^{\*}>0$ under evaluation units are inefficient. In the assessment of efficiency with the directional distance function, there should at least be one inefficient unit. Directional Distance Function (DDF) has the following Specifications [10].

DDF is translation invariant if $g^{+}=g\_{o},$ where $g^{+}$ is distance vector after the transfer of all the data as$ \left(x\_{j}+t^{-} , y\_{j}+t^{+}\right)$ , $ \left(t^{-}, t^{+}\right)\ne 0\_{m+p} $such that the optimal solutions value of model remain without unchanged namely $β\_{o}^{t\*}=β\_{o}^{\*}$ to consider that $β\_{o}^{\*}, $ $β\_{o}^{t\*}$ are optimal solution values of model (2) for evaluation of vectors $\left(x\_{o}, y\_{o}\right)$ and $\left(x\_{o}+t^{-} , y\_{o}+t^{+}\right)$ respectively and image point of vector $\left(x\_{o}+t^{-} , y\_{o}+t^{+}\right)$ where is as $\left(x\_{o}+t^{-} , y\_{o}+t^{+}\right)+ β\_{o}^{t\*}\left(-g\_{o}^{t-}, g\_{o}^{t+}\right)$ ,with image point of vector $\left(x\_{o},y\_{o}\right) $as $\left[\left(x\_{o}, y\_{o}\right)+ β\_{o}^{\*}\left(-g\_{o}^{-}, g\_{o}^{+}\right)\right]+\left(t^{-}, t^{+}\right) $ is same. In other words, vector should not be dependent on sample of units. Directional distance function for constant values has translation invariant specifications. In other words, direction vector should not be dependent on sample of units. DDF for constant values has translation invariant specifications.

**2.3. FDH Model**

Considering the observed DMUs, FDH model [4] is introduced for the calculation of efficiency in the absence of convex situation in production possibility set according to DDF as follows.

$β\_{o}^{\*}=Max $ $ β\_{o}$ $+ϵ \left(\sum\_{k=1}^{p}s\_{k}^{+}+\sum\_{i=1}^{m}s\_{i}^{-}\right)$

$s.t.$ $\sum\_{j=1}^{n}λ\_{j}x\_{ij}+s\_{i}^{-}=x\_{i}^{o}-β\_{o}g\_{io}^{-} $, $i=1,…,m,$

 $\sum\_{j=1}^{n}λ\_{j}y\_{kj}- s\_{k}^{+}=y\_{k}^{o}+β\_{o}g\_{ko}^{+} $, $k=1,…,p,$ (3)

 $\sum\_{j=1}^{n}λ\_{j}=1$, $λ\_{j}\in \left\{0, 1\right\}$ $λ\_{j}\geq 0$, $j=1,…, n.$

Note that the production possibility set is not a convex polytope, like DEA traditional models. If $β\_{o}^{\*}=0$ under evaluation unit are efficient otherwise it is inefficient.

**3. Calculation of Value Efficiency based on Directional Distance Function:**

**3.1. Value efficiency in variable return to scale model:**

Considering variable return to scale technology for the calculation of value efficiency with each unit. In first, we solved model (2) for the unit under assessment and was obtained the optimal solution model (2) and separate efficiency units under model (2). Note that the efficiency value was determined as$ 1-β\_{o}^{\*}$. Note that production possibility set frontier is a convex frontier and production possibility set is convex polytope. Thus, for the approximation of value function frontier passage of MPS is determined in advance and uses all possible tangents. Hamel et al. [8] solved BCC model to achieve VEA value and in the first phase the values of $λ^{\*}=\left(λ\_{1}^{\*}, …, λ\_{n}^{\*}\right)$ were achieved for each unit and in the second phase by freeing variable values of $λ\_{j} $took positive values in the first phase, solved the model again. These corresponding indices of efficient units in the reference set with the unit are under assessment. And the obtained value of efficiency is considered as VEA value. In this study, directional distance function model is used instead of BCC model. In model (2), distance vector should first be selected in various stats. (1) $g^{o}=\left(-x\_{o},0\_{s}\right)$ is considered that BCC model of the input orientation is obtained. (2)$ g^{o}=\left(0\_{m},y\_{o}\right)$ is considered that BCC model of output orientation is obtained. (3) $g^{o}=\left(-x\_{o},y\_{o}\right) $is considered that combined model is achieved. (4) $g\_{o}=\left(-\left(x\_{o}-\overline{x}\right) , \left(\overbar{y}-y\_{o}\right)\right)$ where the ith component of

Minimum$ \overline{x}$ is the input value of ith input in input data set and the rth component of$ \overbar{y} $is the max rth output value in the data set that RDM model is achieved [10]. If we remove the constraint $\sum\_{j=1}^{n}λ\_{j}=1$ then we obtain CCR model in the all case.

In any model, the efficiency value is presented by solving model (2). After solving model (2) and determining the corresponding solution with each unit under evaluation from model (2) and we obtain vector$ λ^{\*}=\left(λ\_{1}^{\*}, …, λ\_{n}^{\*}\right)$ from model (2), in the following we solve the model (4) by attention to obtain vector$ λ^{\*}=\left(λ\_{1}^{\*}, …, λ\_{n}^{\*}\right)$ from model (2). For calculating the value efficiency, the following model is presented.

$Max $ $ β\_{o}$ $+ϵ \left(\sum\_{k=1}^{p}s\_{k}^{+}+\sum\_{i=1}^{m}s\_{i}^{-}\right)$

$s.t.$ $\sum\_{j=1}^{n}λ\_{j}x\_{ij}+s\_{i}^{-}=x\_{i}^{o}-β\_{o}g\_{io}^{-} $, $i=1,…,m,$

 $\sum\_{j=1}^{n}λ\_{j}y\_{kj}- s\_{k}^{+}=y\_{k}^{o}+β\_{o}g\_{ko}^{+} $, $k=1,…,p,$ (4)

 $\sum\_{j=1}^{n}λ\_{j}=1$, $λ\_{j}\geq 0, if λ\_{j}^{\*}=0$, $j=1,…, n,$ $ϵ>0.$

It means that in the above model, after the determination of $ λ^{\*}$ obtained from model (2), $ λ^{\*} $variables which equal zero, are not in the corresponding reference set with the unit under assessment and $1-β\_{o}^{\*}$. Value shows the efficiency of value. However, model (4) can be developed, instead of solving the corresponding model with, reference units in the solution of model (2) use our intended units. We place $λ\_{j}^{\*}>0$ only for a subset of set $N-\left\{λ\_{j}^{\*}>0\right\}$ and limit MPS selection and determine the amount of value efficiency. The above problem is described in the parts of applied numerical example.

**3.2. Value Efficiency in FDH Models:**

The difference between traditional models of DEA and FDH is in production technology. Production possibility set in traditional models of DEA is a convex set. While this is not the case in FDH models. For example, assume the data of Table (1), production possibility set regarding the equal inputs in the output space is as Fig. 2.



Figure 2: The linear value function and the FDH model.

 As you can see, A, B, C and D units are efficient units while these units are inefficient in CCR or BCC models. Because D unit is the combination of E and C units, the efficiency value of D unit in BCC model equals 0.89 < 1.

Note that FDH model produce more or equally optimistic efficiency scores in comparison with DEA traditional models which are greater or equal to this credible issue. Because FDH production possibility set is a subset of production possibility set of traditional models and the values re never worse. While the number of observed units is less compared with the number of inputs and outputs. Efficiency analysis leads to more efficiency units in FDH models compared with traditional models. Now, we intend to introduce superior information in FDH models. As previously discussed, for the calculation of VEA, there is one assumption which is production possibility set is convex to be able to have an unknown value function frontier. Convex production possibility set is convex to enable us to approximate value unknown function frontier by a linear frontier and the approximate obtained VEA is always greater than technical efficiency and the optimistic model occurs. If the production possibility set is not convex, the previously presented linear approximation cannot be used as the value function approximation. VEA calculation method should be corrected because the value function in this point is not at its optimistic value in D point.

Another reason why linear approximation cannot work in this mode except for when the unit under assessment is dominant by convex combining of other units is that linear approximation might be pessimistic. Considering figure (2) and the value function of figure (2), this function achieves its maximum value in B or C points. Assume this point is B. The linear approximation method for the value function with a linear function is not right because the strength is very pessimistic. Consider F unit. The right amount of value efficiency is between its approximate value and FDH efficiency, i.e., 0.81 < 0.83 < 0.93. In this mode, it is possible that the approximation is either very optimistic or very pessimistic. In this regard, the convex cones are used [13]. Assuming that value function is a quasi-concave function and the assumption of quasi-concave is stronger than pseudo-concave and the frontier of value function is approximated by convex cones. And a lower bound of value efficiency values is presented which is under the approximation based on DM information. We necessarily consider a specific MPS and the superiority information based on paired comparisons are obtained. After the assessment of units and comparing them, MPU units are selected. Assume that $f:R^{m+p}\rightarrow R$ function is a quasi-convex function. r is the point $Z\_{1}$, …, $Z\_{r}$ $\in $ $R^{m+p}$ while $Z\_{i}\geq Z\_{k}$ $i=1,…, r $ $i\ne k $; cone $C\left(Z\_{1},…, Z\_{r}:Z\_{k}\right)$ with vertex $Z\_{k}$ is defined as follows.

$$C\left(Z\_{1},…, Z\_{r}:Z\_{k}\right)= \left\{z=z\_{k}+ \sum\_{\begin{array}{c}i=1\\i\ne k\end{array}}^{r}\left(z\_{k}-z\_{i}\right)μ\_{i} , μ\_{i}\geq 0, i=1,…, r, i\ne k \right\} (5)$$

A cone can include one or two points or r points, assuming that $f:R^{m+p}\rightarrow R$ function is quasi-concave, by lemma 1 in [13] we have

$∀ z \in \left(Z\_{1},…, Z\_{r}:Z\_{k}\right)$ $f\left(z\_{i}\right) \geq f\left(z\_{k}\right)\geq f\left(z\right)$ $z\in $ $R^{m+p}, i=1,…, r $ $i\ne k$

For identifying whether $z$ $\in $ $R^{m+p} $is dominated by points cone (6) we can solve model (6).

$$Max ϵ$$

$s.t.$ $z\_{k}+ \sum\_{\begin{array}{c}i=1\\i\ne k\end{array}}^{r}\left(z\_{k}-z\_{i}\right)μ\_{i}-ϵ \geq z$ (6)

 $μ\_{i}\geq 0$,

 If $ε\geq 0$ in the optimal solution, then $ z^{\*}\in \left(Z\_{1},…, Z\_{r}:Z\_{k}\right) $exists in a way that:

$f\left(z\_{i}\right) \geq f\left(z\_{k}\right)\geq f\left(z\right)$,$ z\in R^{m+p} , $ $z^{\*}\geq z$ , $i=1,…, r. $ If$ ε >0 $in the optimal solution, then $f\left(z\_{i}\right) \geq f\left(z\_{k}\right)>f\left(z\right)$.

**Definition 3**.**2.1.** Point $z\in $ $R^{m+p}$ is dominated by point's cone $C\left(Z\_{1},…, Z\_{r}:Z\_{k}\right)$ if point $z^{\*}\in \left(Z\_{1},…, Z\_{r}:Z\_{k}\right)$ in a way in which $z^{\*}\geq z$ , $z^{\*}\ne z$.

**Definition 3**.**2.2.**  Point $z\in $ $R^{m+p}$ is strongly dominated by point's cone $C\left(Z\_{1},…, Z\_{r}:Z\_{k}\right)$ if point $z^{\*}\in \left(Z\_{1},…, Z\_{r}:Z\_{k}\right)$ in a way in which$ z^{\*}>z$.

If in the optimal solution of model (7), $ε>0$, then vector $z$ is strongly dominated by cone $C\left(Z\_{1},…, Z\_{r}:Z\_{k}\right)$ and if $ε=0$, then we will have two changes or $z$ is not dominated by Cone points $C\left(Z\_{1},…, Z\_{r}:Z\_{k}\right)$ or $z$ is dominated but not strongly. Model 6 is corrected as model (7).

$$Max ϵ$$

$s.t.$ $z\_{k}+ \sum\_{\begin{array}{c}i=1\\i\ne k\end{array}}^{r}\left(z\_{k}-z\_{i}\right)μ\_{i}-ϵw\geq z$ (7)

 $μ\_{i}\geq 0$, $w\in $ $R^{m+p}$ , $w\geq 0$ $w\ne 0$.

If in model (6), $ε=0$,, but in model (7), ε > 0, then for some $w\geq 0$ and $w\ne 0$ $z$ is dominated by Cone C.

If amount of $ε$ is unbounded in model (7) and for some w > 0 and w0, then superiority information of $z\_{i}\geq z\_{k}$ $i=1,…, r $ $i\ne k$ would not be compatible (see [9]). We assume the utility function is quasi-concave. If in model (6), ε > 0 but $z$ is not dominated by non-superior cone, then $f\left(z\_{k}\right)>f\left(z\right)$.

 Consider Figure (3) and assume that a unit B is considered as MPU, dominant points like F and G cannot be considered as MPU. We introduce two points' cones $C\left(B,C;C\right) $with by vertex C and lines that pass of C trough B. We can make four relevant two-point cones with by vertex A, C, D and E. Lines that pass of B trough A, C, D and E units are shown in figure 3. G, F and D, E units are dominated by cone $C\left(B,C;C\right)$ where all points of cones $C\left(B,E;E\right)$ and $C\left(B,D;D\right)$ are also dominated. Cones $C\left(B,A;A\right),$ $C\left(B,C;C\right) $include all the information we require. If DM considers B and D units as MPU units, we can produce the three point cone$ C\left(B,C, D;C\right)$. Each point of this cone dominated C unit and some of these points also dominate B and D points.



Figure 3: Analysis units when B is the most preferred unit

The corresponding solution of model (7) is unbounded. Regarding figure (3), there are no quasi-concave function whose optimal solutions are B and D points and thus the unbounded of model (7) shows that superiority information does not exist in accordance with the assumption that value function is pseudo- concave. Now, we can use convex cones approximation to achieve the lower approximation of quasi-concave value function frontier that passes MPU function. Considering $z=\left(-x,y\right),$ $z\_{1}$, …, $z\_{r}$ $\in $ $R^{m+p}$ which is a subset of all points, the unit under evaluation is shown by $z^{0}$. Considering unit G in figure 3, with regard to unknown value function, the right efficiency value for this unit that is $\left|\frac{OG}{oG^{2}}\right|$ is approximated as$ \left|\frac{OG}{oG^{3}}\right| $ and our approximation is less pessimistic because $\left|\frac{OG}{oG^{3}}\right|=0.63 <\left|\frac{OG}{oG^{2}}\right|0.75$. Now, value efficiency evolution model is presented based on FDH model as follows.

$Max $ $β$ $+ϵ \left(\sum\_{j=1}^{p}s\_{j}^{+}+\sum\_{h=1}^{m}s\_{h}^{-}\right)$

$s.t.$ $x\_{kh}+\sum\_{i=1}^{r}μ\_{i}\left(x\_{kh}-x\_{ih}\right)+s\_{h}^{-}=x\_{h}^{o}-β g\_{h}^{-o}$ $h=1,…,m,$

 $y\_{kj}+\sum\_{i=1}^{r}μ\_{i}\left(y\_{kj}-y\_{ij}\right)-s\_{j}^{+}=y\_{j}^{o}+β g\_{j}^{+o}$ , $j=1,…,p,$ (8)

 $s\_{j}^{+}, s\_{h}^{-}\geq 0$, $μ\_{i}\geq 0$, $j=1,…,p$, $h=1,…,m,$ $ϵ>0 Non-archimedean$.

The unit under evaluation is shown by $z^{0}=\left(-x^{o},y^{o}\right). $We can write model (8) as a vector as follows [9].

$Max β$ $+ϵ1^{T}s$

$s.t.$ $z\_{k}+ \sum\_{\begin{array}{c}i=1\\i\ne k\end{array}}^{r}\left(z\_{k}-z\_{i}\right)μ\_{i}-s=βg^{o}+z^{0}$ (9)

 $μ\_{i}\geq 0$, $w\in $ $R^{m+p}$ , $w\geq 0$ $w\ne 0$.

**4. Value Efficiency Analysis based on Bank Branches:**

In this section, we calculated value efficiency analysis of bank branches based on model with constant and variable return scale and models without the condition of being convex or FDH. In this regard, to calculate the efficiency of branches of Helsinki metropo- litan bank [9], the presented models in this paper were used. Branches of Helsinki metropo- litan bank are owned by OP-Pohjola Group and continue group service in Finland. Banks present financing, investment, daily and insurance services for private customers and small businesses. Note that bank services are important channels for the presentation of sale performance on banks and management seeks opportunities to improve sale performance in the branches. This issue is discussed in problem is also discussed in Eskelinen, Halme and Kallio [6] as well as in Eskelinen and Kuosmanen [7] papers from various aspects. Sale performance of a branch is defined as below and the amount sale of non-produced is by its sale force. Bank management seeks to identify units with weaker performances to improve them. Inputs and outputs are considered as below. Outputs include sales volume of transactions from branches. Sales are in two classes. The first class is financial services and Daily Banking investment services of funds services are excluded from our analyses. Insurance services have entered investments but non-life insurance services are not considered. The weights of importance of factors for the development of output are determined through management. Only one input is considered in our analyses. A sale force management does not consider other operational costs due to the fact that they are not controllable. The input quality used is the overall use of work times in sales activity as full-time equivalents. First, the value efficiency was calculated using CCR, BCC, RDM and FDH models. Then, the value efficiency of branches of banks is assessed with the opinion of management of banks. Thus, MPU unit is specified and other units should achieve the levels of their activity of MPU units input and output levels. The input and output data are given in Table (2). The results are given in Table (2).

Table 2: Input and outputs

|  |  |  |  |
| --- | --- | --- | --- |
| DMU | $$O\_{1}$$ | $$O\_{2}$$ | $$I\_{1}$$ |
| 1 | 1090 | 497 | 26 |
| 2 | 2633 | 1111 | 47.7 |
| 3 | 3320 | 1477 | 60.7 |
| 4 | 1147 | 353 | 25.2 |
| 5 | 1180 | 540 | 21.6 |
| 6 | 3821 | 1769 | 75.5 |
| 7 | 1574 | 716 | 36.4 |
| 8 | 1171 | 1004 | 29.1 |
| 9 | 1174 | 449 | 22.5 |
| 10 | 1203 | 568 | 27.2 |
| 11 | 928 | 384 | 22 |
| 12 | 4393 | 2210 | 65.9 |
| 13 | 2642 | 931 | 38.8 |
| 14 | 3362 | 1505 | 53.1 |
| 15 | 2263 | 541 | 26.9 |
| 16 | 3619 | 1541 | 70.3 |
| 17 | 4163 | 1594 | 73.6 |
| 18 | 3075 | 805 | 46.7 |
| 19 | 5757 | 2601 | 93 |
| 20 | 1763 | 496 | 29 |
| 21 | 3825 | 1961 | 83.1 |
| 22 | 2354 | 792 | 42.4 |
| 23 | 5289 | 3160 | 104 |
| 24 | 1108 | 332 | 24.2 |
| 25 | 743 | 354 | 22.4 |

In first, we will evaluate the value and traditional efficiency of 25 bank branch using models CCR and BCC in the input orientation. First, consider the second and third columns of the Table (3). As it can be seen, the units 8, 12 and 15 of the CCR efficiency are the input orientation. With regarding the units 12 and 15 as branches that have the best performance from management's perspective, by using the model (4) and to consider;$ g=\left(g\_{x},g\_{y}\right)=\left(-x,0\right)$, and With the removal of restrictions $\sum\_{j=1}^{n}λ\_{j}=1$ , we will obtain the value proficiency rates of bank branches in the input orientation. The third column of the Table (3) shows value proficiency rates, as it can be seen only the units of 12 and 15 are the value efficiency.

The rates of the value and traditional proficiency of CCR are distinct in the input orientation of units 8, 21 and 23, for example, the traditional efficiency of unit 21 is equal to 0.703, while the rate of the value efficiency of CCR input orientation corresponding with it is 0.696, which is less than own normal value. Unit 8 is the traditional efficiency of CCR, while the rate of its value efficiency is 0.772 and it is not the value efficient. Now we consider the rates of the value and traditional efficiency resulted of BCC model in input orientation.

To solve the model (2) vector, we must consider;$ g=\left(g\_{x},g\_{y}\right)=\left(-x\_{o},0\right)$. By regarding the third and fourth columns of Table (3), we will see that units 5, 8, 12, 15, 19 and 23 are BCC efficiency of input orientation. By regarding units 12 and 15 as the MPU units, it has been come the value efficiency of BCC input orientation in the last column Table (3), as it can be seen Units 1, 4, 5, 7, 9, 10, 11, 13, 14, 19, 20, 23, 24 and 25 have the distinct rates of the value and traditional efficiency of BCC in input orientation. For example, the rates of the value and traditional efficiency of BCC of the input orientation of the unit 9 are 0.96, 0.879 respectively. Units 5, 19 and 23 are BCC Efficient while the value efficient is not BCC.

Table 3: Efficiency and value efficiency scores of the CCR and BCC input orientation models.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| DMU | CCR-input-o- efficiency | CCR-input –o-value-efficiency | BCC-input –o-efficiency | BCC-input –o-value-efficiency |
| DMU01 | 0.606 | 0.606 |  0.831  | 0.775 |
| DMU02 | 0.775 | 0.775 |  0.805  | 0.805 |
| DMU03 | 0.783 | 0.783 |  0.792  | 0.792 |
| DMU04 | 0.578 | 0.578 |  0.857  | 0.715 |
| DMU05 | 0.790 | 0.790 |  1.000  | 0.988 |
| DMU06 | 0.735 | 0.735 |  0.736  | 0.736 |
| DMU07 | 0.624 | 0.624 |  0.725  | 0.723 |
| DMU08 | 1.000 | 0.772 |  1.000  | 1.000 |
| DMU09 | 0.708 | 0.708 |  0.960  | 0.879 |
| DMU10 | 0.647 | 0.647 |  0.815  | 0.806 |
| DMU11 | 0.588 | 0.588 |  0.982  | 0.792 |
| DMU12 | 1.000 | 1.000 |  1.000  | 1.000 |
| DMU13 | 0.900 | 0.900 |  0.913  | 0.913 |
| DMU14 | 0.908 | 0.908 |  0.918  | 0.918 |
| DMU15 | 1.000 | 1.000 |  1.000  | 1.000 |
| DMU16 | 0.725 | 0.725 |  0.736  | 0.736 |
| DMU17 | 0.768 | 0.768 |  0.838  | 0.838 |
| DMU18 | 0.800 | 0.800 |  0.894  | 0.894 |
| DMU19 | 0.891 | 0.891 |  1.000  | 0.977 |
| DMU20 | 0.753 | 0.753 |  0.843  | 0.813 |
| DMU21 | 0.703 | 0.696 |  0.708  | 0.708 |
| DMU22 | 0.724 | 0.724 |  0.745  | 0.745 |
| DMU23 | 0.896 | 0.820 |  1.000  | 0.832 |
| DMU24 | 0.577 | 0.577 |  0.893  | 0.722 |
| DMU25 | 0.487 | 0.487 |  0.964  | 0.713 |

Table (4) shows the efficiency values and the value efficiency of CCR and BCC models in the output orientation. First, consider the efficiency values of the output orientation of CCR. According to the second and third columns of the Table (4), units 8, 12 and 15 are CCR efficient of the output orientation. With regarding the units 12 and 15 as MPU of the value efficiency rates in the third column of Table (4). As it can be seen, the only units 12 and 15 are the value efficient and the Amounts of the value and traditional efficiency of CCR of the output-orientation of the units 8, 21, 23 are distinct.

For example, the value efficiency rates of CCR of the output orientation of unite 21 are 0.703 and 0.696, respectively. The third and fourth columns of Table (4) show the value and traditional efficiency rates of CCR in the output orientation. Units 5, 8, 12, 15, 19 and 23 are BCC efficient units of the output orientation. With regarding units 12 and 15 as MPU units and resolving the model (4) with the vector selection of $g=\left(g\_{x},g\_{y}\right)=\left(0,y\_{o}\right)$ will see that the only units 8, 12 and 15 values are the value efficient of the output orientation of BCC.

 The amounts of the value and traditional efficiency of the output orientation of BCC of units 1, 4, 5, 6, 9, 10, 11, 16, 17, 19, 21, 23, 24 and 25 are according to the last column of Table (4) are distinct. For example, the rates of the value and traditional efficiency of the output orientation of BCC of unit 11 are 0.710 and 0.735, respectively that the amounts of the value efficiency are smaller. Units 5, 19 and 23 are BCC efficient of the output-orientation, while the value efficient isn’t the output-orientation of BCC.

Table 4: Efficiency and value efficiency scores of the CCR and BCC output orientation models.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| DMU | CCR-output-o- efficiency | CCR-output –o- value-efficiency | BCC-output-o- efficiency | BCC-output –o-value-efficiency |
| DMU01 | 0.606 | 0.606 | 0.708 | 0.705 |
| DMU02 | 0.775 | 0.775 | 0.776 | 0.776 |
| DMU03 | 0.783 | 0.783 | 0.808 | 0.808 |
| DMU04 | 0.578 | 0.578 | 0.625 | 0.622 |
| DMU05 | 0.790 | 0.790 | 1.000 | 0.983 |
| DMU06 | 0.735 | 0.735 | 0.784 | 0.777 |
| DMU07 | 0.624 | 0.624 | 0.666 | 0.666 |
| DMU08 | 1.000 | 0.772 | 1.000 | 1.000 |
| DMU09 | 0.708 | 0.708 | 0.861 | 0.833 |
| DMU10 | 0.647 | 0.647 | 0.750 | 0.749 |
| DMU11 | 0.588 | 0.588 | 0.735 | 0.710 |
| DMU12 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU13 | 0.900 | 0.900 | 0.907 | 0.907 |
| DMU14 | 0.908 | 0.908 | 0.910 | 0.910 |
| DMU15 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU16 | 0.725 | 0.725 | 0.784 | 0.781 |
| DMU17 | 0.768 | 0.768 | 0.871 | 0.865 |
| DMU18 | 0.800 | 0.800 | 0.919 | 0.919 |
| DMU19 | 0.891 | 0.891 | 1.000 | 0.980 |
| DMU20 | 0.753 | 0.753 | 0.762 | 0.762 |
| DMU21 | 0.703 | 0.696 | 0.766 | 0.717 |
| DMU22 | 0.724 | 0.724 | 0.757 | 0.757 |
| DMU23 | 0.896 | 0.820 | 1.000 | 0.821 |
| DMU24 | 0.577 | 0.577 | 0.647 | 0.626 |
| DMU25 | 0.487 | 0.487 | 0.612 | 0.603 |

Table (5) shows the efficiency and the value efficiency scores obtained from the mix orientation models. To calculate the value efficiency, unites 12 and 15 are considered as MPUs, units that have the best performance from management's perspective. First, we will consider CCR model. And it is selected the vector of$ g=\left(g\_{x},g\_{y}\right)=\left(-x\_{o},y\_{o}\right)$. As it can be seen in the second column of Table (5), units of 8, 12 and 15 are the mix efficiency units of CCR.

Only units 12 and 15 are the mix value efficient units of CCR. Amounts of the value and traditional efficiency of mix CCR of units 8, 21 and 23 are distinct according to the second and third columns in Table (5). All value efficiency amounts are less than the same amounts to the traditional efficiency. For example, the mix efficacy of the value and traditional CCR of unit 21 are 0.825 and 0.821, respectively that the amount of the value efficiency is smaller.

Unit 8 is CCR mix efficiency, while it is not the mix value efficient and its value efficient CCR is equal to 0.871.

The fourth and fifth columns of Table (5) show the rates of the mix value and traditional efficiency of the BCC model (technology of variable returns to scale). Units 12 and 15 are included as MPU.

 As it is seen, units 8, 12 and 15 are the mix efficiency units of BCC and other units are inefficient. The mix value efficiency amount of BCC of all units except units 8, 12 and 15 with their corresponding values means the mix traditional efficiency of BCC in accordance the fourth and fifth columns of Table (5) are distinct, for example, the amount of the usual mix value and traditional efficiency of unit 18 is 0.889 and 0.952, respectively.

The amounts of value efficiency of BCC mix orientation of all units are larger than its same values means the mix traditional efficiency of BCC. As it was observed, by applying the management opinion for that the performances of all units are such as units with the best performance, it can be used the concept of the value efficiency.

Table 5: Efficiency and value efficiency scores of the mix orientation models.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| DMU | CCR-Mix-o- efficiency | CCR-Mix-o- value-efficiency | BCC-Mix-o- efficiency | BCC- Mix-o- value-efficiency |
| DMU01 | 0.754 | 0.754 | 0.754 | 0.854 |
| DMU02 | 0.873 | 0.873 | 0.873 | 0.883 |
| DMU03 | 0.878 | 0.878 | 0.878 | 0.881 |
| DMU04 | 0.733 | 0.733 | 0.733 | 0.806 |
| DMU05 | 0.883 | 0.883 | 0.883 | 0.993 |
| DMU06 | 0.847 | 0.847 | 0.847 | 0.862 |
| DMU07 | 0.769 | 0.769 | 0.769 | 0.822 |
| DMU08 | 1.000 | 0.871 | 1.000 | 1.000 |
| DMU09 | 0.829 | 0.829 | 0.829 | 0.924 |
| DMU10 | 0.786 | 0.786 | 0.786 | 0.877 |
| DMU11 | 0.741 | 0.741 | 0.741 | 0.862 |
| DMU12 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU13 | 0.948 | 0.948 | 0.948 | 0.950 |
| DMU14 | 0.952 | 0.952 | 0.952 | 0.955 |
| DMU15 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU16 | 0.841 | 0.841 | 0.841 | 0.864 |
| DMU17 | 0.869 | 0.869 | 0.869 | 0.920 |
| DMU18 | 0.889 | 0.889 | 0.889 | 0.952 |
| DMU19 | 0.942 | 0.942 | 0.942 | 0.989 |
| DMU20 | 0.859 | 0.859 | 0.859 | 0.883 |
| DMU21 | 0.825 | 0.821 | 0.825 | 0.821 |
| DMU22 | 0.840 | 0.840 | 0.840 | 0.841 |
| DMU23 | 0.945 | 0.901 | 0.945 | 0.905 |
| DMU24 | 0.732 | 0.732 | 0.732 | 0.810 |
| DMU25 | 0.655 | 0.655 | 0.655 | 0.800 |

In this section we will evaluate the efficiency of the input and output orientation using the FDH model. By considering the second column of Table (6), we will see that from 25 units, 12 ones are FDH efficient units. Others units are inefficient. The sale network management distinguishes units 12, 13 and 15 as the best branches in terms the performance and we will respect them as MPUs units and obtain the efficiency values on the basis of these units.

 As we saw, Unit 13 isn’t efficient based on CCR and BCC models and it is efficient in FDH model. By considering the mix convex assume in the collection, it will be defeated the production possibility of this unit using a convex combination of other units. To use the model (8) to evaluate the value efficiency, we will use four-point cones that all other efficiency units except the three units can be used as the vertices of the cone. This means that we use any other efficient unit as the victor of the cone and use the three units as the other components of the four-point cone. Three units 12, 13 and 15 are considered as units with having the best performance in terms of management. The resolution of the model (8) for each of the above cones of units 2 and 23 which were diagnosed with FDH efficient model aren’t ineffective with value efficient model (8) any more. And, they obtain levels of 0.984 and 0.872, respectively, as the amount of value efficiency in the input orientation. This indicates that the value efficiency scores will be different with the traditional efficiency scores by mentioning the considered value information. The fourth and fifth columns of the Table (6) are the representation of the value and traditional efficiency rates of FDH ​​in the output orientation, respectively.

 As can be seen, units 2 and 5, 7, 8, 10, 12, 13, 14, 15, 18, 19 and 23 are the FDH efficient of the output orientation and other units are inefficient. By considering the same cones with the input orientation, it is shown the results related to the value efficiency amounts of FDH output orientation in the last column of the Table (6). Units 2 and 23 are like FDH input efficient but they aren’t FDH value efficient. Levels of value efficiency obtained from the model resolution (8) for units 2 and 3, 6, 21 and 33 are distinct from the FDH traditional efficiency amount of their output orientation, which are marked in Table (6) with red, for example, it is 0.87 and 0.852, respectively, the value and traditional efficiency amounts of the output orientation of unit 6 that the value efficiency amount is smaller.

 For the value and traditional efficiency amounts of the input orientation in accordance with the second column of the Table (6) we have also similar interpretation. For example, unit 6 has the traditional-value efficiency of the input orientation as 0.873 and 0.825, respectively that the value efficiency the input orientation amount of FDH is less than the amount corresponding to the value efficiency the input orientation. It should be noticed that unit 25 due to being new, it was not considered as MPU unit. Unit 10 also has the same performance with unit 15 and the service amount produces the similar investment and nearly 50 percent of finance services less than the unit 15.

Table 6: Efficiency and value efficiency scores of the FDH input and output orientation models.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| DMU | FDH- input-o- efficiency | FDH- input –o-value-efficiency | FDH-Output –o-efficiency | FDH-value- Output–o-efficiency |
| DMU01 | 0.831 | 0.831 | 0.924 | 0.924 |
| DMU02 | 1.000 | 0.984 | 1.000 | 0.917 |
| DMU03 | 0.875 | 0.781 | 0.988 | 0.887 |
| DMU04 | 0.857 | 0.857 | 0.972 | 0.972 |
| DMU05 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU06 | 0.873 | 0.825 | 0.870 | 0.852 |
| DMU07 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU08 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU09 | 0.960 | 0.960 | 0.995 | 0.995 |
| DMU10 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU11 | 0.982 | 0.982 | 0.786 | 0.786 |
| DMU12 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU13 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU14 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU15 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU16 | 0.937 | 0.937 | 0.824 | 0.824 |
| DMU17 | 0.895 | 0.895 | 0.948 | 0.948 |
| DMU18 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU19 | 1.000 | 1.000 | 1.000 | 1.000 |
| DMU20 | 0.928 | 0.928 | 0.917 | 0.917 |
| DMU21 | 0.793 | 0.677 | 0.887 | 0.786 |
| DMU22 | 0.915 | 0.915 | 0.891 | 0.891 |
| DMU23 | 1.000 | 0.872 | 1.000 | 0.892 |
| DMU24 | 0.893 | 0.893 | 0.939 | 0.939 |
| DMU25 | 0.964 | 0.964 | 0.656 | 0.656 |

**5. Conclusions**

In this paper we discussed about the set calculation of the value efficiency from the decision-making units in different technologies. In this regard, we used the DEA models based on the directional distance function. By offering a single model of rates, we obtained the concept of efficiency. As it can be seen, it can be considered the value information and importance of those unites that have the best performance in terms management as the evaluation and ranking criteria of other units. As it was observed, based on the existing units, even units that are a convex combination of other units can obtain the value efficiency. All provided models obtain the value efficiency in all technologies including the return to variable, mix and constant scale and constant and FDH o with different orientation.

In the study, we applied proposed models for 25 bank branches and obtained the values of efficiency in different technologies. By applying the bank's management opinion about that 3 units have the best performance, we obtained the efficiency of other units and their ranking based on these units and we determined that some of the units efficient in ordinary technologies, while they are not value efficiency. Finally, it can be developed above models for inaccurate data and the network models of DEA.

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