

Equivalent HPM with ADM and Convergence of the HPM to a Class of Nonlinear Integral Equations

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Abstract. The purpose of this study is to implement homotopy perturbation method, for solving nonlinear Volterra integral equations. In this work, a reliable approach for convergence of the HPM when applied to a class of nonlinear Volterra integral equations is discussed. Convergence analysis is reliable enough to estimate the maximum absolute truncated error of the series solution. The results obtained by using HPM, are compared to those obtained by using Adomian decomposition method alone. The numerical results, demonstrate that HPM technique, gives the approximate solution with faster convergence rate and higher accuracy than using the standard ADM.

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1. Introduction

In the recent years, the homotopy perturbation method (HPM), has been applied to a wide class of engineering and scientific problems and in many interesting mathematics and physics areas. Some important problems in sciences and engineering can usually be reduced to a system

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of integral equations. Several powerful methods have been proposed to obtain exact solutions of nonlinear partial differential equations, such as the decomposition method [1, 2, 3], the $(\frac{G'}{G})$ -expansion method [4], the Laplace Adomian decomposition method [5], differential transform method [6], the homotopy perturbation method [7, 8, 9], the homotopy analysis method [10, 11], the Exp-function method [12, 13]. The homotopy perturbation method (HPM) has some significant advantages over numerical methods. It provides analytic, verifiable, rapidly convergent approximations which yield insight into the character and the behavior of the solution just as in the closed form solution. The homotopy perturbation method, gives a reasonable basis for studying linear and nonlinear system of integral equations. The homotopy perturbation method (HPM) solves successfully different types of linear and nonlinear equations in engineering and sciences. The application of homotopy perturbation method used as an alternative solution method to a wide variety of integral problems. The method is applicable to integral problems that can be reduced to a finite set of non-linear (or linear) integral equations. While the applicability of the method to the problem of HPM to different types of integral equations has been discussed by many authors, for example [9]. In this work, the nonlinear Volterra integral equation of the second kind

$$u(t) = g(t) + \int_0^t K(t, \xi) f(u(\xi)) d\xi, \quad (1)$$

is considered where $g(t)$ is assumed to be bounded $\forall t \in J = [0, T]$ and $|k(t, \xi)| \leq M, \forall 0 \leq \xi \leq T$. The nonlinear term $f(u)$ is Lipschitz continuous with $|f(u) - f(v)| \leq L|y - z|$ and has the polynomial representation

$$B_n = N(S_n) - \sum_{j=0}^{n-1} B_j, \quad (2)$$

where the partial sum is $S_n = \sum_{i=0}^n u_i$.

$$u = \sum_{i=0}^{\infty} u_i, \quad (3)$$

where

$$N(u) = - \int_0^t K(t, \xi) f(u(\xi)) d\xi, \quad (4)$$

$$f(u) = \sum_{n=0}^{\infty} A_n, \quad (5)$$

where A_n is Adomian polynomial. Thus we have

$$N\left(\sum_{n=0}^{\infty} u_n\right) = - \sum_{i=0}^{\infty} \int_0^t K(t, \xi) A_i d\xi = - \sum_{i=0}^{\infty} B_i, \quad (6)$$

where by applying homotopy perturbation method we get

$$u_0(t) = g(t), \quad (7)$$

$$u_k(t) = B_{k-1}, \quad k \geq 1. \quad (8)$$

But by using of Adomian polynomial B_n define as follows

$$f(u) = \sum_{n=0}^{\infty} A_n, \quad (9)$$

$$B_n = \int_0^t K(t, \xi) A_n d\xi = \frac{1}{n!} \frac{d^n}{d\lambda^n} \int_0^t K(t, \xi) f(u(\lambda)) \Big|_{\lambda=0} d\xi. \quad (10)$$

In the present work, we study and prove the equivalent homotopy perturbation method with Adomian decomposition method and bring several theorems. The aim of the present paper implement and prove equivalent homotopy perturbation method with Adomian decomposition method and convergence homotopy perturbation method for a class of nonlinear Volterra integral equations. The paper is organized as follows: In Section 2, brief discussions for homotopy perturbation method are presented and approximate solution is obtained. In Section 3, we describe the equivalent HPM with ADM briefly and apply this technique with a simple example. Section 4, contains convergence analysis using of the several theorems. In Section 5, contains numerical results. Also a conclusion is given in Section 6. Finally some references are given at the end of this paper.

2. Homotopy Perturbation Method

In this section, the homotopy perturbation method (HPM) is presented. Instead of ordinary perturbation methods, this method doesn't need a small parameter in an equation. According to this method, a homotopy with an embedding parameter $p \in [0, 1]$ is constructed and the embedding parameter is considered as a "small parameter". Thus, this method is called the homotopy perturbation method. HPM is a powerful tool for solving various nonlinear equations, especially nonlinear partial differential equations. Recently this method has attracted a wide class of audience in all fields of science and engineering. This method proposed by a Chinese mathematician J.H. He [18]. In this investigation; HPM is used to obtain the numerical solution of the nonlinear Volterra integral equations. The numerical solutions which are found are compared with the exact solutions. To illustrate the basic idea of the homotopy perturbation method, consider the following nonlinear equation

$$A(v) - f(r) = 0, \quad r \in \Omega, \quad (11)$$

subject to the boundary condition:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma, \quad (12)$$

where A is written as follows:

$$A(v) = L(v) + N(v), \quad r \in \Omega. \quad (13)$$

Here A is a general differential operator, B is a boundary operator, $f(r)$ is known analytic function, Γ is the boundary of the domain Ω and $\frac{\partial}{\partial n}$ denotes differentiation along with the normal vector drawn outwards Ω . The operator A can generally be divided into two parts L , N. So, Eq. (11) can be rewritten as follows :

$$L(v) + N(v) - f(r) = 0. \quad (14)$$

By the homotopy technique, He [18] constructed a homotopy $v(r, p) : \Omega \times [0, 1] \rightarrow \mathbb{R}$ which satisfies

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \quad p \in [0, 1], \quad r \in \Omega, \quad (15)$$

where $p \in [0, 1]$ is an embedding operator, and u_0 is an initial approximation of Eq. (15), which satisfies the boundary condition Eq. (40). Obviously, we have

$$H(v, 0) = L(v) - L(u_0) = 0, \quad H(v, 1) = A(v) - f(r) = 0. \quad (16)$$

The change process of p from zero to unity is just that of $v(r, p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopic. We consider v as following:

$$v = v_0 + v_1p + v_2p^2 + v_3p^3 + \dots \quad (17)$$

According to HPM, the best approximate solution of Eq. (14) can be explained as a series of powers of p ,

$$u = \lim_{p \rightarrow 1} v = \sum_{k=0}^{\infty} v_k = v_0 + v_1 + v_2 + \dots \quad (18)$$

The above convergence is given [16].

3. Equivalent HPM with ADM

Let L operator has inverse, then L^{-1} there exist

$$L^{-1}L(v) = L^{-1}0 + L^{-1}L(u_0) - pL^{-1}L(u_0) - pL^{-1}[N(v) - f(r)], \quad (19)$$

suppose $L^{-1}0 = \Phi$, then we have

$$v = \Phi + u_0 - pu_0 - pL^{-1}[N(v) - f(r)], \quad (20)$$

$$\sum_{k=0}^{\infty} v_k p^k = \Phi + u_0 - pu_0 - pL^{-1}[\sum_{k=0}^{\infty} Nv_k p^k - f(r)]. \quad (21)$$

If $Lu_0 \neq 0$, then we obtain

$$v_0 = \Phi + u_0, \quad (22)$$

$$v_1 = -u_0 - L^{-1}[Nv_0 - f(r)],$$

$$v_2 = -L^{-1}Nv_1,$$

$$\vdots$$

$$v_{k+1} = -L^{-1}Nv_k, \quad \forall k = 2, 3, \dots$$

Now if $Lu_0 = 0$, we get

$$v_0 = u_0, \tag{23}$$

$$v_1 = -L^{-1}[Nv_0 - f(r)],$$

$$v_2 = -L^{-1}Nv_1,$$

$$\vdots$$

$$v_{k+1} = -L^{-1}Nv_k, \quad \forall k = 2, 3, \dots$$

One can see that homotopy perturbation method equivalent with Adomian decomposition method. As a simple example, consider the nonlinear initial value problem

$$\frac{du}{dx} = u^2, \tag{24}$$

with the initial condition $u(0) = 1$. This differential equation has the exact solution of $y(x) = \frac{1}{1-x}$. By using of the homotopy perturbation method we are going to $v_0 = u_0 = 1$. Following the method described above, we define a linear operator $L = \frac{d}{dx}$. The inverse operator is then

$$L^{-1} = \int_0^x (\cdot) dx. \tag{25}$$

Because $L, Nv = -v^2 = -\sum_{i=0}^{\infty} \sum_{j=0}^i v_j v_{i-j}$ and $f(r) = 0$, given then we can determine the recursive relationship that will be used to generate the solution

$$v_0 = 1, \tag{26}$$

$$v_{n+1} = -L^{-1}Nv_n, \quad \forall n = 0, 1, \dots$$

$$v_1 = -L^{-1}[Nv_0] = L^{-1}(1) = x, \tag{27}$$

$$v_2 = -L^{-1}Nv_1 = L^{-1}(2v_0v_1) = x^2,$$

$$v_3 = -L^{-1}Nv_2 = L^{-1}(2v_0v_2 + v_1^2) = x^3,$$

and so on. Then we get

$$u = \lim_{p \rightarrow 1} v = \lim_{p \rightarrow 1} \sum_{k=0}^{\infty} v_k p^k = \sum_{k=0}^{\infty} v_k = 1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}, \quad (28)$$

where is exact solution. Now applying Adomian decomposition method. Rewriting the differential equation Eq. (24) in operator form, we have $Lu = Nu$, where N is a nonlinear operator such that $Nu = u^2$. Next we apply the inverse operator for L to the equation. On the left hand side of the equation, this gives

$$L^{-1}Lu = u(x) - u(0). \quad (29)$$

Using the initial condition, this becomes

$$L^{-1}Lu = u(x) - 1. \quad (30)$$

Returning this to equation $Nu = u^2$, we now have

$$u(x) = 1 + L^{-1}(Nu). \quad (31)$$

Next, we need to generate the Adomian polynomials, A_n . Let u be expanded as an infinite series $y(t) = \sum_{n=0}^{\infty} u_n(t)$ and define $Ny = \sum_{n=0}^{\infty} A_n$. Then

$$\sum_{n=0}^{\infty} u_n(t) = 1 + L^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \quad (32)$$

To find A_n , we introduce the scalar λ such that

$$u(\lambda) = \sum_{n=0}^{\infty} \lambda^n u_n. \quad (33)$$

Then

$$Nu(\lambda) = \sum_{n=0}^{\infty} \lambda^n \sum_{i=0}^n (u_i u_{n-i}). \quad (34)$$

From the definition of the Adomian polynomials

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} (\text{Nu}(\lambda)) \Big|_{\lambda=0}, \quad (35)$$

we find the Adomian polynomials

$$A_0 = u_0^2, \quad (36)$$

$$A_1 = 2u_0u_1,$$

$$A_2 = 2u_0u_2 + u_1^2,$$

$$A_3 = 2u_0u_3 + 2u_1u_2,$$

and so on. Returning the Adomian polynomials to equation Eq. (32), we can determine the recursive relationship that will be used to generate the solution.

$$u_0(x) = 1, \quad (37)$$

$$u_{n+1}(x) = L^{-1}A_n, \quad \forall n = 0, 1, \dots$$

Solving this yields

$$u_0(x) = 1, \quad (38)$$

$$u_1(x) = x,$$

$$u_2(x) = x^2,$$

and so on. We can see that the series solution generated by this method is

$$u(x) = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n, \quad (39)$$

which we recognize as the Taylor series for the exact solution

$$u(x) = \frac{1}{1-x}. \quad (40)$$

4. Application of ADM and HPM for Nonlinear Voltera Equations

Example 4.1. Consider the following nonlinear Volterra integro-differential equation

$$u'(x) = \frac{x^5}{5} - \int_0^t ((u^2(t) - 2)dt), \quad u(0) = 0, \quad (41)$$

with the exact solution $u(x) = x^2$. Proceeding as before, we apply both of the methods on problem (43). We summarize the errors of both methods for various values of x in the following table. The errors for methods ADM and HPM are alike.

ADM and HPM:

Commencing with $u(0) = u_0 = 0$, and with equating coefficients of like powers of p , we obtain

$$u_0(x) = 0, \quad (42)$$

$$u_1(x) = 0.3333333333x^6 + x^2,$$

$$u_2(x) = 0,$$

$$u_3(x) = -.6105006104 \times 10^{-5} \times x^{14} - .0007407407407x^{10} - .03333333333x^6,$$

$$u_4(x) = 0,$$

$$u_5(x) = .8809532617 \times 10^{-9} \times x^{22} + .2012834257 \times 10^{-6} \times x^{18}$$

$$+.00002035002035x^{14} + .0007407407407x^{10},$$

Table 1: The absolute error, between the exact solution (43) and the numerical solution.

x_i	(one iteration)	(two iterations)	(three iterations)	(four iterations)
0.0	0.1000×10^{-8}	0.1000×10^{-8}	0.2000×10^{-8}	0.0000
0.1	0.0049	0.0050	0.0050	0.0050
0.2	0.0193	0.0200	0.0200	0.0200
0.3	0.0415	0.0451	0.0453	0.0453
0.4	0.0693	0.0799	0.0810	0.0811

Example 4.2. Let us second consider the nonlinear Volterra integral equation

$$u(x) = \exp(x) - \frac{1}{3} \exp(3x) + \frac{1}{3} + \int_0^x u^3(t)dt, \quad u(0) = 1, \quad (43)$$

with the exact solution $u(x) = \exp(x)$.

ADM and HPM:

Commencing with $u(0) = u_0 = \exp(x)$, and with equating coefficients of like powers of p , we obtain

$$u_0(x) = \exp(x), \quad (44)$$

$$u_1(x) = 0,$$

$$u_{n+1}(x) = 0, \text{ for } n \geq 1.$$

On the other hand, the exact solution

$$u(x) = \exp(x).$$

Example 4.3. We consider the nonlinear Volterra integral equation as follows

$$u(x) = \frac{1}{6}(-3 + 8\cos(x) + \cos(2x)) + \int_0^x \sin(x-t)(1 + u^2(t))dt, \quad (45)$$

with the initial condition $u_0(x) = \frac{1}{6}(-3 + 8\cos(x) + \cos(2x))$.

ADM and HPM:

Commencing with $u(0) = u_0$, approximate solution is obtained as follows

$$\begin{aligned} u_{\text{app}}(x) = \sum_{k=0}^4 u_k(x) = & .0003741114852\cos(x)^{13} + .9612424460\cos(x) + .1666666667\cos(2x) \\ & - 2.861667685\cos(x)^2 + 3.647977656\cos(x)^3 - 4.545313212\cos(x)^4 + \\ & .00001336112446\cos(x)^{14} + 3.968139001\cos(x)^5 - 1.698544428\cos(x)^6 + \\ & .2915487624\cos(x)^7 + .01521000719\cos(x)^8 - .09754755734\cos(x)^9 + \\ & .01461863391\cos(x)^{10} + .01813055105\cos(x)^{11} + .004004286116\cos(x)^{12} + 1.115147401, \end{aligned} \quad (46)$$

Table 2: The absolute error, between the exact solution (45) and the numerical solution.

x_i	(one iteration)	(two iterations)	(three iterations)	(four iterations)
0.0	0.1000×10^{-8}	0.1000×10^{-8}	0.2000×10^{-8}	0.0000
0.1	0.0049	0.0050	0.0050	0.0050
0.2	0.0193	0.0200	0.0200	0.0200
0.3	0.0415	0.0451	0.0453	0.0453
0.4	0.0693	0.0799	0.0810	0.0811

5. Convergence Analysis

5.1. Uniqueness theorem

Theorem 5.1. *Problem (1) has a unique solution whenever $0 < \alpha < 1$, where, $\alpha = LMT$.*

Proof. Let u and u^* be two different solutions to Eq. (1) then

$$\begin{aligned}
 |u - u^*| &= |Nu - Nu^*| = \left| \int_0^t K(t, z)(f(u) - f(u^*)) dz \right| \\
 &\leq \int_0^t |K(t, z)| |f(u) - f(u^*)| dz \\
 &\leq LM \int_0^t |u - u^*| dz \\
 &\leq LMT |u - u^*| = \alpha |u - u^*|
 \end{aligned}$$

from which we get $(1 - \alpha)|u - u^*| \leq 0$. Since $0 < \alpha < 1$, then $|u - u^*| = 0$, implies $u = u^*$ and this completes the proof. \square

Theorem 5.2. *The series solution (3) of problem (1) using of the HPM converges if $0 < \alpha < 1$ and $|u_1| < 1$.*

Proof. Denote as $(C[J], \|\cdot\|)$ the Banach space of all continuous functions on J with the norm $\|f(t)\| = \max_{t \in J} |f(t)|$. Define the sequence of partial sums S_n ; let S_n and S_m be arbitrary partial sums with $n \geq m$.

We are going to prove that S_n is a Cauchy sequence in this Banach space: With notice to relations $S_n = \sum_{i=0}^n u_i$ and (2) we have

$$\begin{aligned}
\|S_n - S_m\| &= \max_{\forall t \in J} |S_n - S_m| & (47) \\
&= \max_{\forall t \in J} \left| \sum_{i=m+1}^n u_i \right| = \max_{\forall t \in J} \left| \sum_{i=m}^{n-1} N u_i \right| = \max_{\forall t \in J} \left| \sum_{i=m}^{n-1} B_i \right| \\
&= \max_{\forall t \in J} \left| \sum_{i=m}^{n-1} \int_0^t K(t, z) A_i dz \right| = \max_{\forall t \in J} \left| \int_0^t K(t, z) \sum_{i=m}^{n-1} A_i dz \right| \\
&= \max_{\forall t \in J} \left| \int_0^t K(t, z) (f(S_{n-1}) - f(S_{m-1})) dz \right| \\
&\leq \max_{\forall t \in J} \int_0^t |K(t, z)| |f(S_{n-1}) - f(S_{m-1})| dz \\
&\leq \text{LMT} \|S_{n-1} - S_{m-1}\| = \alpha \|S_{n-1} - S_{m-1}\|
\end{aligned}$$

Let $n = m + 1$; then

$$\|S_{m+1} - S_m\| \leq \alpha \|S_m - S_{m-1}\| \leq \alpha^2 \|S_{m-1} - S_{m-2}\| \leq \dots \leq \alpha^m \|S_1 - S_0\|. \quad (48)$$

From the triangle inequality we have

$$\begin{aligned}
\|S_n - S_m\| &\leq \|S_{m+1} - S_m\| + \|S_{m+2} - S_{m+1}\| + \dots + \|S_n - S_{n-1}\| & (49) \\
&\leq [\alpha^m + \alpha^{m+1} + \dots + \alpha^{n-1}] \|S_1 - S_0\| \\
&\leq \alpha^m [1 + \alpha + \alpha^2 + \dots + \alpha^{n-m-1}] \|S_1 - S_0\| \\
&\leq \alpha^m \left(\frac{1 - \alpha^{n-m}}{1 - \alpha} \right) \|u_1(t)\|.
\end{aligned}$$

Since $0 < \alpha < 1$ we have $(1 - \alpha^{n-m}) < 1$; then

$$\|S_n - S_m\| \leq \frac{1 - \alpha^{n-m}}{1 - \alpha} \max_{\forall t \in J} |u_1(t)|. \quad (50)$$

But $|u_1| < 1$ (since $g(t)$ is bounded); so, as $m \rightarrow 1$, then $\|S_n - S_m\| \rightarrow 0$. We conclude that S_n is a Cauchy sequence in $C[J]$, so the series converges and the proof is complete. \square

5.2. Error Estimate

Theorem 5.3. *The maximum absolute truncation error of the series solution (3) to problem (1) is estimated to be*
 $\max_{\forall t \in J} |u(t) - \sum_{i=0}^m u_i(t)| \leq \frac{K\alpha^{m+1}}{L(1-\alpha)}$ where $K = \max_{\forall t \in J} |f(g(t))|$.

Proof. From Theorem 5.2, inequality (50) we have

$$\|S_n - S_m\| \leq \frac{1 - \alpha^{n-m}}{1 - \alpha} \max_{\forall t \in J} |u_1(t)|. \quad (51)$$

As $n \rightarrow \infty$ then $S_n \rightarrow u(t)$ and $\max_{\forall t \in J} |u_1(t)| \leq TM \max_{\forall t \in J} |f(u_0)|$, so

$$\|u(t) - S_m\| \leq \frac{\alpha^{m+1}}{L(1-\alpha)} \max_{\forall t \in J} |f(g(t))|. \quad (52)$$

Therefore, the maximum absolute truncation error in the interval J is

$$\max_{\forall t \in J} |u(t) - \sum_{i=0}^m | \leq \frac{K\alpha^{m+1}}{L(1-\alpha)}. \quad (53)$$

This completes the proof. \square

6. Numerical Results

Formula (2) can be converged faster of Formula (10). For example, if $f(y) = y^2$ the first four polynomials using Formulas (2) and (10) are computed to be:

Applying Formula (10) we obtain:

$$B_0 = \int_0^t K(t, z) u_0^2 dz, \quad (54)$$

$$B_1 = \int_0^t K(t, z) 2u_0 u_1 dz, \quad (55)$$

$$B_2 = \int_0^t K(t, z) (u_1^2 + 2u_0 u_2) dz, \quad (56)$$

$$B_3 = \int_0^t K(t, z) (2u_1 u_2 + 2u_0 u_3) dz. \quad (57)$$

Applying Formula (2) we obtain :

$$B_0 = \int_0^t K(t, z) u_0^2 dz, \quad (58)$$

$$B_1 = \int_0^t K(t, z) (u_1^2 + 2u_0 u_1) dz, \quad (59)$$

$$B_2 = \int_0^t K(t, z) (u_2^2 + 2u_0 u_2 + 2u_1 u_2) dz, \quad (60)$$

$$B_3 = \int_0^t K(t, z) (u_3^2 + 2u_0 u_3 + 2u_1 u_3 + 2u_2 u_3) dz. \quad (61)$$

Clearly, the first four polynomials computed using Formula (2) include the first four polynomials computed using Formula (10) in addition to other terms which should appear in B_4, B_5, B_6, \dots using Formula (10). Thus, the solution using Formula (2) forces many terms to be entered into the calculation processes earlier, yielding a faster convergence. In order to verify the conclusions of Theorems 5.2 and 5.3, consider the following numerical example:

$$u(t) = \frac{1}{20}(300 + 315t^2 + 5t^4 + t^6) - \frac{1}{150} \int_0^t (t - \xi) u^2(\xi) d\xi, \quad 0 \leq t \leq 1,$$

with exact solution $u(t) = 15(1 + t^2)$. Table 1, shows the exact absolute truncation error

$\Lambda = |u(t) - \sum_{k=0}^m u_k(t)|_{t=1}$ and the maximum absolute truncation error $\Lambda^* = \frac{K\alpha^{m+1}}{L(1-\alpha)}$ for different values of m where $T = 1$, $M = \frac{1}{150}$, $L = 60$, $\alpha = \frac{2}{5}$ and $K = \frac{385641}{400}$. Table 1 shows the exact absolute truncation error and the maximum absolute truncation error.

Table 3: shows the exact absolute truncation error and the maximum absolute truncation error.

m	$\Lambda = u(t) - \sum_{k=0}^m u_k(t) _{t=1}$	$\Lambda^* = \frac{K\alpha^{m+1}}{L(1-\alpha)}$
5	1.23333×10^{-3}	0.109693
10	6.27671×10^{-9}	0.00112326
15	2.60089×10^{-14}	0.0000115022
20	9.79976×10^{-19}	1.17782×10^{-7}

7. Conclusion

In this work, the convergence of homotopy perturbation method, as applied to solving a class of Volterra integral equations, has been thoroughly investigated. We have proved several theorems for Volterra nonlinear integral equations. Our numerical experiment confirms the convergence of the method. It also shows that the homotopy perturbation method is the same with Adomian decomposition method.

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